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THE MATHEMATICS TEACHER

Volume XLIV MAY · 1951

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THE MATHEMATICS TEACHER

Volume XLIV Number 5

An Arithmetic Bulletin for Parents

By Elizabeth J. Roudebush
Director of Mathematics, Seattle Public Schools, Seattle, Washington

At the suggestion of the Superintendent of Schools, the curriculum staff of the Seattle Public Schools has initiated a plan to inform parents of the ways by which their children learn the three R's. Four committees were formed to devise bulletins describing the methods for teaching arithmetic, reading, spelling and writing in the Seattle schools. The arithmetic bulletin given in this article was sent with the report card to the parents of each child in our elementary schools in November, 1950.

The preparation of the bulletin was a major task. A committee of six curriculum consultants and assistants first set down the questions that parents ask most frequently. Then answers were written in simple direct language. Every effort was made to avoid theoretical expressions or pedagogical lingo. The questions and answers were then studied by the entire curriculum staff. This group suggested many changes, additions and deletions. The committee, the Mathematics Director, and the Mathematics Assistant served as an editorial group to formulate a revised edition. This copy was again studied and improved by the curriculum staff.

The next step was to send copies of the material in mimeographed form to every elementary school. Principals and teachers studied the text and sent in many comments and some suggestions for further change. The committee incorporated these suggestions from the teachers in the final bulletin.

The consultant in charge of publications, the Director of Art, and the Director of Mathematics worked together in planning the format, drawing the sketches, laying out the copy, and supervising the printing of the bulletin.

Parent reaction to the bulletin was good. The most frequent comments were, "This is the kind of information we like to have the schools furnish," and "There should be more of this type of bulletin." Elementary teachers and principals have shown increased interest in the arithmetic program since the bulletin was issued.

How We Teach Arithmetic

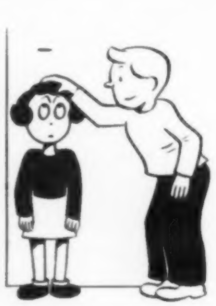
When does your child begin to learn arithmetic? When does your child get

specific drill in arithmetic? How is your child taught subtraction? Do we teach the multiplication tables? Which comes first—long or short division? When are fractions taught? Why is your child often asked to estimate an answer before working a problem? Of what value are story problems? Should your child "prove" problems? Who checks the arithmetic papers? Do we require homework? What is "good work" in arithmetic? How can parents help?

The Answers . . .

When does your child begin to learn arithmetic?

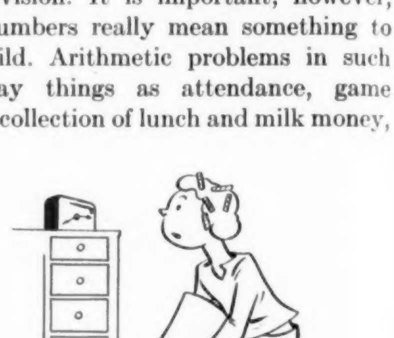
The use of arithmetic begins in the home when your child knows how old he is, when he can tell coins apart, or when



he recognizes that he is taller than Mary but shorter than Jim. In the kindergarten he probably learns to count objects from one to twenty, to recognize small groups of objects, and to know the meanings of such words as *few, many, up, down, thick, thin, short,* and *tall.* The actual reading and writing of numbers begins in the first grade.

When does your child get specific drill in arithmetic?

Specific drill begins in the second grade after your child learns to add and subtract through the use of actual objects such as books, pencils, and money. Drill speeds up addition, subtraction, multiplication, and division. It is important, however, that numbers really mean something to the child. Arithmetic problems in such everyday things as attendance, game scores, collection of lunch and milk money,



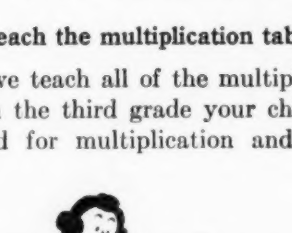
school banking, and telling time provide real experience with numbers. Then we give our practice problems.

How is your child taught subtraction?

The need for subtraction arises when your child meets a question such as "How many are left?" or "How many more?" With pictures, objects, stories, and dramatizations your child takes a smaller group from a larger group and finds how many remain. The "take-away-borrowing" method of subtraction is used with numbers of two or more figures. However, if your child has learned some other method of subtraction, he is not asked to change.

Do we teach the multiplication tables?

Yes, we teach all of the multiplication facts. In the third grade your child sees the need for multiplication and learns



some of the facts. By the time he has finished the fourth grade he probably will know "nine times nine." Understanding is most important. Instead of teaching "three times five" first, Johnny decides that a family of five needs fifteen sandwiches if each one is to have three on a picnic. Practice with multiplication tables continues through junior high school and even into senior high school if necessary.

Which comes first—long or short division?

Long division with a single-figure divisor comes first because through that longer method your child can follow each step as he thinks it through. He may use short division as soon as he is sure of these steps. Short division is first presented in the fifth grade, retaught in the sixth grade, and used extensively in the seventh grade. Short division, after all, is merely a short cut.

When are fractions taught?

The use of fractions occurs in every grade. Your child begins using fractions

when he does such things as distributing or sharing materials, and folding paper into equal parts. In grade five he begins adding and subtracting fractions by using parts of real objects before he adds and subtracts with fractional numbers. By the end of the sixth grade he will have had multiplication and division of fractions and some preliminary work with decimals. In the seventh grade the emphasis is on decimal fractions and percentage.

Why is your child often asked to estimate an answer before working a problem?

Your child ought to know if his answer makes sense. Since estimating helps him to recognize a reasonable answer, approximating is encouraged. Estimating is not a substitute for working the problem. Its purpose is to develop the child's judgment.

Of what value are story problems?

Your child uses numbers many times during the day—in keeping scores, play-

ing, buying, and banking. It is on these real needs that his arithmetic work is based. He is more interested in learning and remembering number facts when he uses them in connection with his work and play. Story problems help him see how important addition, subtraction, multiplication, and division are to him.

Should your child "prove" problems?

"Proving" the answer to the problem affords the child a feeling of success. The amount of written "proving" that each child needs to do is determined by the skill and accuracy with which he works.

Who checks the arithmetic papers?

There are a number of ways of checking errors on arithmetic papers. For example, a paper may be checked by the child himself, by another child, by a committee, or by the teacher and the child. The teacher makes the final check after the child has corrected his errors.

Do we require homework?

Homework is not often required. Usually enough work time is provided during the school day. When homework is necessary, the teacher plans it with the child.

What is "good work" in arithmetic?

Understanding of arithmetic comes to different children in different ways and at different ages. In each grade, material previously presented is retaught because all children do not master the fundamentals of arithmetic at exactly the same time. However, each child should work up to the level of his ability. "Good work" in arithmetic means the development of systematic, orderly habits of accomplishment as well as the mastery of the fundamental skills and understandings.

How can parents help?

Parents can best help by including the child in the many daily situations in which

numbers are used such as shopping, measuring, counting, sharing, budgeting, banking, and playing games. Number games such as dominoes or "bean bags" are fun for the whole family and will improve your child's skill in arithmetic. Often a family experience will provide a background for an original story problem which the child will enjoy presenting to his class.

Mathematically Trained Personnel Needed in the Government Service

By Daniel B. Lloyd

Department of Mathematics, Wilson Teachers College, Washington, D. C.

In the present era of national mobilization and rearmament the problem of personnel selection for civilian federal employment becomes increasingly important. It is proposed to discuss herein the latest trends in employment policies and practices in government service with emphasis on implications for teachers and counselors in mathematics. The main source of the data was the United States Civil Service Commission which holds examinations in hundreds of cities throughout the country, covering jobs in all grades and classifications. Information desired concerning a particular kind of job, or examination therefore, should be requested from them in Washington, D.C., or from one of their other 13 regional offices.

The author has interviewed a large number of government officials and specialists responsible for the preparation of civil service examinations. The view prevails among them that a larger proportion of questions involving mathematics, and the training that mathematics provides, will be included in the examinations. This applies in general to all types of

examinations, not merely to those of a mathematical nature. The reason for this is that precision techniques or precision instruments are being utilized increasingly in all lines of work, and the need for precision workers is likewise increasing.

More and more the examinations will require precision and accuracy in computations, in reading of verbal problems, in interpretation of meaning, in translation from verbal to mathematical symbols, and setting down in clear and methodical form the given data. The ability to reason logically and systematically in problems and problem situations, ability to observe relations existing in or deducible from the data, and good judgment and skill in the choice and use of techniques applicable to their solution, will be searchingly tested. Officials asserted that those applicants having "blind spots" in mathematics will not be wanted in government work. Skill in mathematical techniques they regarded as an index of their judgment and clear thinking in the widest possible range of positions, whether technical, clerical, or skilled and semi-skilled trades.

Teachers and employers, in the opinion of Civil Service officials, are coming more and more to a common belief that thorough mathematical training is essential to success and advancement in a wide field of occupations and professions.

Civil Service announcements are periodically issued calling for applicants for a wide range of trade jobs, from air-conditioning mechanic to awning maker, and from machinist to tailor. For all of these, at least accuracy in arithmetic is needed, and higher mathematical proficiency definitely improves the applicant's chances for appointment. The greatest present demand for tradesmen is in the building trades,—electricians, plumbers, carpenters, and painters.

Junior engineers, scientists, and technicians in many fields are also in frequent demand. Mathematics and science loom as vital needs for these along with special training or experience, or both, in the specific field chosen. Sub-professional grades, such as scientific aide or technician, requires no college training, but placement on the list of eligibles is on a relative basis and the higher the qualifications the better the applicant's chances for appointment. Graduation from high school in a course including three credits in mathematics and science is evaluated as a year's practical experience.

On the college level there is a present demand for physicists, mathematicians, electronic scientists, draftsmen, and metallurgists. Announcement or examination for other scientific fields is also made periodically. Formal graduation from college is no longer *technically* required, but training or experience *equivalent* thereto is expected.

Many young people with business training enter the government service through junior commercial positions, such as typist, stenographer, clerk, and bookkeeper. Computational skill, reading of tables, use of simple formulas and geometry, as well as specialized skill in the particular field, are essentials for such work. Such beginning jobs are frequently opening wedges to higher administrative positions.

Mathematics used in government service may be considered as of two types: (1) that used by technical workers, including the professional mathematician; and (2) that used by the "Jack of all Trades" who uses any mathematics appropriate to his field of work. The sub-professional grades generally use arithmetic, algebra, geometry and sometimes trigonometry. The professional grades might use mathematics through the calculus, and other higher branches depending on their particular specialty. A survey of a few of the bureaus show that the Naval Observatory uses astronomy and spherical trigonometry; the Geodetic Survey uses map projection, least squares, probability, and complex variable analysis; the Bureau of Standards uses quantum mechanics, differential equations, and various branches of mathematical physics.

Trades and Allied Positions

Positions ranging from unskilled laborer to the highly specialized trades constitute a wide range of opportunity. Outside of Washington far more of these positions are available than any other type. Most persons enter as trade apprentices and advance through successive grades of their chosen trade. The apprenticeship program has been increasing for some time. The Government Printing Office has long had an apprenticeship program for bookbinders, machinists, electricians, compositors, printers, and others. The Navy Yards and other agencies of the Department of Defense, such as Air Force and Army Ordnance, offer apprenticeships for boat builders, shipfitters, coppersmiths, molders, pipefitters, toolmakers, machinists, instrument makers, sheet metal workers, joiners, pattern makers, and electricians.

The apprenticeship usually lasts four years and includes evening classes in related theory as well as a wide and varied program of job experience. After satisfactory service in this, the apprentice becomes a journeyman, i.e., an experienced "regular" in the trade. The minimum entrance age for apprentices is 16, and a regular journeyman 18. Some journeymen enter the government service, also, without further apprenticeship. They are not given an entrance test, but submit their work experience record.

All apprentice tests include arithmetic and practical science. The arithmetic test emphasizes fundamental operations with fractions and decimals (division by decimals is a stumbling block for many); operations with denominate numbers, and their application to measurement problems; use of the metric system; conversion of units; and percentage. The science questions cover simple machines, such as gears, pulleys, levers, as well as automobile repairs, and miscellaneous mechanical information. Formerly pictures were used in this section of the test, but now most of the questions are in the form of verbal descriptions and instructions.

In the algebra, geometry, and trigonometry questions, which are required for the more technical trades, a broad coverage of fundamentals is required; a few fractional exponents, as well as radicals; equations and formulas, solving for any letter; variation and proportion; and verbal problems of the simpler type, are included.

The entire composition of the test battery is as follows:

Subject	No. of Questions
Arithmetic	60
Practical Science	50
Algebra	40
Geometry	15
Form Board	25
U. S. History	15
English Usage (grammar and spelling)	35

Applicants have shown much more weakness in mathematics than in science. Those having had geometry excel on the Form Board test. Scores are superior from areas where teachers are best paid. The validity of this test, as measured by correlation with supervisors' subsequent ratings on the job, is highest with the science section, and next highest with the mathematics.

The government wishes all apprentices trained in algebra, as is required by large private concerns, but according to officials does not find enough so trained. They say that a boy going into apprentice work having had trigonometry would be successful to an extent comparable with a man entering the business world 50 years ago with a college degree.

Sub-professional Positions

These positions are open to those having an elementary knowledge of some specialized professional field. Appointees are assigned to office, field, or laboratory to assist in carrying on certain professional work. Typical of these are:

1. Engineering Draftsman
2. Engineering Aide
3. Scientific Aide, with options in
   a. Chemistry
   b. Geology
   c. Mathematics

d. Metallurgy
e. Meteorology
f. Physics
g. Radio
4. Medical Technician
5. Graduate Nurse
6. Staff Dietician
7. Student Dietician
8. Occupational Therapy Aide
9. Social Case Worker
10. Library Assistant
11. Agricultural Aide

Adequate training of a specified amount in the option selected is necessary. College or specialized technical training is often helpful to qualify. Mathematics on a level useful to the field of work is required.

As these positions can lead the appointee to higher professional positions, it is important that the beginner shall have had all college preparatory mathematics, i.e., trigonometry, solid geometry, and college algebra. While on the job he can then take college or other technical training on a part-time study basis, thus equipping himself for more rapid upgrading.

A typical test battery for this range of positions is composed as follows:

(1) Elementary science
(2) Mathematics, including 20% algebra, and the rest computations, interpreting and reading of tables and graphs.
(3) Scale, gauge, and instrument reading.
(4) Spatial visualization and perception.
(5) Vocabulary.

Mathematics is the common core in all such technical examinations as well as in general aptitude type questions and in some specialized questions in the particular field. Sample examination questions for the positions sought can be secured from the Commission on request.

A high school graduate would possibly rate GS-1 on the civil service pay classification schedule. This was formerly called SP-2 (Sub-professional 2). He could ultimately rise to GS-7, commensurate in pay with the beginning professional grades. (*GS* means General Schedule.)

Certain technical agencies have openings at times for *Trainees* who later become eligible for regular positions.

Professional Positions

A college degree, or its equivalent in training or experience is required for the professional grades, starting with GS-5 on the pay schedule. Over half of the professional positions are filled in this lowest professional grade. Promotion to GS-14 may ultimately be attained while in service. The highest grades GS-14 to 18 are limited by legislation. Fewer than 10% of government employees now actually make more than $6400 annually (Maximum of GS-11 grade).

Experience in Sub-professional positions does not in itself qualify for promotions to professional status, except in unusual cases. However, additional night school or extension courses on the college level, leading to a degree, will enable employees to attain professional status. Thus, the Scientific Aide in Mathematics could attain the professional grade of Junior Mathematician.

Professional scientific positions, besides mathematician, include physicist, chemist, electronic scientist, metallurgist, geologist, meteorologist, astronomer, statistician, architect, geographer, geophysicist, engineer (various branches), physician, analyst and technologist in various fields, and in social sciences such as law, economics, library service, teaching (Indian Service only), business management, and so on. Thorough written examinations in the content of the particular field are given for the junior positions. Applicants for the higher grades (GS-7 and above) are rated only on credentials of training and experience submitted.

Last year's announcement for Mathematician is quoted below, in summary:

Mathematician ($3825–$10,000; grades GS-7 GS-15) (no written examination)

Optional Fields Administrative, Applied, Theoretical, General.

Requirements: (A) a bachelor's degree including 24 semester hours in mathematics, and 12 hours in engineering or the physical sciences; or, (B) four years of progressive experience of an appropriate and valuable quality; or (C) an equivalent combination of (A) and (B). In addition, practical ex-

perience of from one to four years is required, of progressive breadth and degree of responsibility for appointment to the higher grades.

Description of Work: Mathematicians appointed to these positions will plan, direct, perform, or assist in performing (a) research in basic mathematical theory or related theoretical analytic or evaluation studies; or, (b) mathematical calculations and computations incident to investigative, developmental and research work in the scientific fields, such as engineering, physics, astronomy, etc. Their duties will include mathematical research, mathematical analyses of observational data, computation of scientific tables, preparation of graphs and charts, and the writing of scientific reports, all involving a thorough knowledge of basic mathematics and in most cases involving a familiarity with the physical sciences or with engineering practices. The difficulty of the work performed and the responsibility assumed will vary with the grade of the position.

Typical duties: 1. Applying methods of mathematical analysis to problems relating to chemistry and physics, including least squares reduction of data; solving equations relating to heat conduction, electrical circuits, chemical reading and stress analyses; making power and Fourier series analyses of empirical curves; computing magnetic fields of models to obtain fits with observed fields; reviewing, analyzing and assisting in the mathematical formulation of physical problems arising in other government agencies or in industry.

2. Planning and initiating research in theory of pure and applied mathematics aimed primarily at developing methods of analysis which will permit the most efficient and general use of high speed automatic electronic computing machinery; studying and formulating requirements for internal organization of such machinery; developing performance specifications for such machinery.

3. Making necessary computations in reduction of data from tide and current records and correlation of resulting data; computing tidal and current differences and constants, and harmonic constants for prediction.

4. Planning and initiating fundamental theoretical investigations in trajectory theory, aerodynamics and hydrodynamics of missiles and aircraft, the effect of control surfaces on the motion of long-range and high velocity guided missiles and aircraft, and advanced theories of optics and radar as applied to ballistic instrumentation.

5. Computing, adjusting by the method of least squares and making analyses of geodetic observations consisting of those for triangulation, leveling and astronomy; computing and making analyses of observation made for the determination of the intensity of gravity and of the isostatic reduction of gravity.

General Data

Some short-cuts are now being used to expedite examining procedure in certain areas, due to urgent needs. More examinations are unassembled and even when written, emphasis is shifted from subject matter content to practical intelligence and general aptitude for job performance. Work habits and skills, keenness of observation, and perception are emphasized.

At the present time, appointments are being made on an "indefinite" basis, due to unsettled conditions. Many of these positions will become permanent in the future, however. Information on current openings can be obtained from the U. S. Civil Service Commission in Washington, D. C. Sample questions for various examinations are available. Also Pamphlet 11, May 1948, "Specimen Questions from U.S. Civil Service Examinations" is on sale for 20¢ from the U. S. Government Printing Office, Washington, D. C. "The Government Employees Exchange" published bi-weekly, 10¢ a copy, at 1740 K Street, N. W., Washington, D. C., is a privately published news bulletin of government jobs. It is an unofficial, but fairly accurate journal. Announcements of openings are posted in postoffices, first and second class, throughout the country.

ANNUAL N.C.T.M. SUMMER CONFERENCE WITH N.E.A.

San Francisco, California, July 2, 1951

Theme: *Improving the Teaching of Mathematics to Meet the Emergency*

See announcement of program on page 329 of the April issue.

Luncheon Reservations: ($2.25 each) should be mailed to Dr. A. J. Hall, San Francisco State College, San Francisco, Calif., not later than June 15, 1951.

Functional Mathematics—Grades Seven Through Twelve*

By William A. Gager

College of Arts and Sciences, University of Florida, Gainesville, Florida

In November 1947 the Florida State Department of Education and the college of Arts and Sciences at the University of Florida sponsored a study under my direction to determine ways of improving certain parts of the present secondary mathematics curriculum. Thirty-six secondary mathematics teachers, representing all areas of subject matter, all types of schools, and all sections of the state were selected to make the study. Work on the project was begun at the University of Florida on June 14, 1948.

Impressed by the surprisingly large number of Courses of Study already available on traditional secondary mathematics the Workshop Group thought it wise not to concern itself with further study of the traditional courses. Instead the Group set as its objectives:

1. To isolate from the traditional mathematics courses, and other sources, those mathematical concepts and principles most essential for effective living in this modern age.
2. To select content materials of interest and value to the respective age groups but which definitely involve the essential mathematical concepts and principles.
3. To organize the materials into a sequence of, functionally worthwhile, mathematically sound courses for grades seven through twelve.

On August 5, 1949, this project was brought to a close and the materials were submitted to the State Department of Education. After a careful study and upon the recommendation of the Florida Courses of Study Committee the State Department of Education placed its stamp of approval upon the study and authorized its publication under the title of *Functional Mathematics in the Secondary Schools*—Bulletin No. 36.

With the basic concepts considered to be of first importance, the Workshop Group set as its first assignment the task of isolating the concepts which it proposed to include in the sequence of functional courses. As one might expect, the personnel of the Group found it very baffling to react concisely and precisely to ideas such as angle, area, equation, locus, similarity, slope, and volume. The fascination of the job however more than compensated for the difficulties encountered.

After numerous revisions there was common agreement that the following 61 concepts would provide adequate stability and flexibility for the proposed functional courses. The list shows that only 35 of the 61 concepts are to be taught in seventh grade and that some of these have been taught in earlier grades; the eighth grade covers 44 concepts with only 9 new ones; the ninth grade 55 concepts with 11 new ones; the tenth grade 60 concepts with only 5 new ones; and the eleventh and twelfth grades 61 concepts with but one new one.

With the realization that concepts by themselves were not enough, the next step taken was the preparation of a list of basic working principles through which the concepts could function. These principles as listed below are those which each student must understand if he hopes to find any meaning in the mathematical processes which follow. One might add that there is little hope that the student will understand these principles in a way that he can effectively use them unless each teacher of mathematics is continuously alert to them and really knows how to teach them for understanding.

* Presented at the Twenty-Eighth Annual Meeting of the National Council of Teachers of Mathematics, Chicago, April 15, 1950.

Basic Concepts	Grades—Seven Through Twelve					
I. Concept of Number						
a. One to one correspondence	7	8	9	10	11	12
b. Digit	7	8	9	10	11	12
c. Integer (whole number)	7	8	9	10	11	12
d. Zero	7	8	9	10	11	12
e. Common fraction	7	8	9	10	11	12
f. Decimal fraction	7	8	9	10	11	12
g. Mixed number	7	8	9	10	11	12
h. Denominate number	7	8	9	10	11	12
i. Abstract number	7	8	9	10	11	12
j. Approximate number	7	8	9	10	11	12
k. Rounded number	7	8	9	10	11	12
l. Exact number	7	8	9	10	11	12
m. Prime number	7	8	9	10	11	12
n. Composite number		8	9	10	11	12
o. Literal number		8	9	10	11	12
p. Signed or directed number		8	9	10	11	12
q. Rational number			9	10	11	12
r. Irrational number			9	10	11	12
s. Imaginary number				10	11	12
t. Complex number				10	11	12
II. Concepts Basic to Operations						
a. Sum	7	8	9	10	11	12
b. Difference	7	8	9	10	11	12
c. Product	7	8	9	10	11	12
d. Quotient	7	8	9	10	11	12
e. Exponent		8	9	10	11	12
f. Logarithm					11	12
III. Concepts of Per Cent and Percentage	7	8	9	10	11	12
IV. Concepts Basic to Measurement						
a. Point	7	8	9	10	11	12
b. Line	7	8	9	10	11	12
c. Angle	7	8	9	10	11	12
d. Plane	7	8	9	10	11	12
e. Plane figure	7	8	9	10	11	12
f. Solid figure	7	8	9	10	11	12
V. Concepts of Measurement						
a. Length	7	8	9	10	11	12
b. Directed magnitude			9	10	11	12
c. Area	7	8	9	10	11	12
d. Volume	7	8	9	10	11	12
e. Angular measure	7	8	9	10	11	12
f. Weight	7	8	9	10	11	12
g. Time	7	8	9	10	11	12
h. Money	7	8	9	10	11	12
i. Heat			9	10	11	12
j. Velocity			9	10	11	12
k. Acceleration			9	10	11	12
l. Power		8	9	10	11	12
VI. Concepts of Functional Relationship						
a. Graph	7	8	9	10	11	12
b. Equation		8	9	10	11	12
c. Series				10	11	12
d. Trigonometric ratio			9	10	11	12
e. Slope				10	11	12
f. Variation			9	10	11	12
VII. Concepts of Comparison						
a. Ratio and proportion	7	8	9	10	11	12
b. Similarity			9	10	11	12
c. Congruency		8	9	10	11	12
d. Symmetry	7	8	9	10	11	12
e. Inequality			9	10	11	12
f. Equivalency		8	9	10	11	12
g. Statistical average	7	8	9	10	11	12
VIII. Concept of Locus		8	9	10	11	12
IX. Concept of Limit			9	10	11	12
X. Concept of Infinity				10	11	12
Totals	35	44	55	60	61	61

BASIC PRINCIPLES

I. In *integers and decimal fractions* the position of the digits in a number determine the value of the number.
   1. The name given to a decimal fraction is determined by the place value of the last digit of the number.
   2. The value of a decimal fraction is not changed by annexing or removing zeros at the right end of the number.
   3. Integers and decimal fractions should be carried out one more place than is desired in an approximate answer.

II. The numerator and denominator of a *common fraction* may be multiplied by the same number (except zero) without changing the value of the fraction.
   1. In final form common fractions may be reduced to lowest form.
   2. Of the three signs, sign of the numerator, sign of the denominator, and the sign before the fraction, any two may be changed without changing the value of the fraction.

III. In *common and decimal fractions* the same quantity may be expressed as a common fraction, as a decimal fraction, and as a per cent of a number.

IV. *Numbers* can be combined by addition, subtraction, multiplication and division.
   1. Like quantities can be added or subtracted. The sum or difference of unlike quantities can be indicated only.
      a. In finding the sums and differences of decimal fractions, the digits of the same place value must be added or subtracted.
      b. Common fractions can be added or subtracted only if they have a common denominator.
      c. Quantities expressed in the same unit of measure can be added or subtracted.
   2. The numbers to be added or multiplied may be combined in any order without changing the result. (Commutative Law)
   3. The numbers to be added or multiplied may be grouped in any manner without changing the value of the result. (Associative Law)
   4. Multiplication may be performed by separating one of the factors into any number of parts, multiplying each part by the other factor and adding the partial products. (Distributive Law)
   5. If one factor of a product is multiplied or divided by a number, the product is multiplied or divided by that number.
   6. If a product is zero, then one or more of its factors is zero.

V. The following *order* is recommended *for performing all operations:*
   1. Perform all operations indicated in the parentheses.
   2. Perform the indicated multiplications and divisions, before the additions and subtractions, in the order in which they occur.
   3. Perform the additions and subtractions in the order in which they occur.

VI. A *ratio* may be expressed as a common fraction, a decimal fraction, and as a per cent.
   1. Per cent should be changed to fractional form before multiplying or dividing.

VII. *Measurement* is the comparison of any magnitude with a unit of measure by which it can be measured.
   1. All measurements are approximations.
   2. The largest allowable error in measurement is one-half the unit of measure used.
   3. Measurements must be expressed in the same unit of measure before computations can be made.
   4. In addition and subtraction of approximate data the result should claim no greater precision than the least precise unit used.
   5. The accuracy of a product or a quotient is no greater than the accuracy of the least accurate factor involved.

VIII. A *quantity* is *equal* to, *greater than,* or *less than* another quantity of the same kind.
   1. A quantity may be substituted for an equal quantity in an expression without changing the value of the expression.
      a. Corresponding parts of congruent figures are equal.
      b. Corresponding angles of similar figures are equal.
   2. If three quantities are so related that the first is greater than the second and the second is greater than the third, then the first is greater than the third.
   3. The whole is equal to the sum of its parts and is greater than either of them.

IX. *Distance* between two points implies shortest distance.
   1. In a plane or space a straight line is the shortest distance between two points.
   2. The perpendicular is the shortest distance from a point to a line or to a plane, or between two parallel lines or two parallel planes.
   3. The minor arc of a great circle is the least distance between two points on a sphere.
   4. The square on the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

X. When one quantity depends upon another for its value the two quantities are said to be *functionally related.*

XI. An *equation* is a statement that two expressions are equal.
   1. The degree of an equation is expressed

by the highest sum of the exponents in any term of an algebraic equation in standard form.
2. An "n^{th}" degree equation has "n" roots.

XII. The same *operation* may be performed *on* both members of an *equation* without changing the balance.
1. If both members of an equation are increased, decreased, multiplied or divided by the same number, or equal numbers, the results are equal.
2. Like powers or roots of both members of an equation are equal.

XIII. To every pair of *rectangular coordinates* there corresponds one and only one point in a plane, and, to every point in a plane there corresponds one and only one pair of rectangular coordinates, with respect to a fixed pair of axes.

XIV. *Comparison* between quantities may be expressed by subtraction or division.
1. Only like quantities can be compared.
2. Ratio is a comparison by division, and is an abstract number.
3. The ratio of the circumference of a circle to its diameter is constant.
4. The ratio of the diagonal of a square to its side is constant.
5. In similar figures the perimeters are in the same ratio as any corresponding sides, the areas as the square of any corresponding sides, and the volumes as the cube of any corresponding sides.

XV. The idea of *locus* involves two interpretations: (a) locus as the path of a moving point so as to satisfy certain given conditions; (b) locus as the composite of many fixed points which satisfy certain given conditions.
1. All points on a locus satisfy the given conditions.
2. All points that satisfy the given conditions are on the locus.

XVI. When the variable V approaches the constant C so that their difference V-C becomes and remains less than any positive number, no matter how small, named in advance, then the variable V is said to approach the constant C as a *limit*.
1. If a variable V approaches a limit C, and each is multiplied by K, then KV approaches KC as a limit.
2. If two variables are always equal while approaching their limits, then the two limits are equal.

XVII. Mathematical *infinity* is the concept of becoming large beyond any known bound and may be represented by the symbol ∞.
1. Division by zero is impossible. However it is sometimes expressed symbolically as $\text{limit}_{x\to 0}\frac{1}{x}=\infty$, where ∞ is understood to represent an idea but not a number.

In order to have some assurance that the concepts and principles herein listed were involved in the proposed materials, so that there would be an acceptable vertical spread of them within each grade and a horizontal unfolding of their meaning from grade to grade, a Grade Placement Chart was prepared to cover the six grades.

The Workshop Group, in selecting materials for functional courses for seventh and eighth grades, did not greatly alter the present offerings. Keeping in mind the varied interests of seventh and eighth graders and the fact that their mathematical needs are actually very few, much effort was expended to build the content around pupil interests and to introduce new concepts and review old ones through activities. It was hoped that this approach would permit the pupils to work with the materials in such ways that they might feel the elation that could come from their own discovery of workable procedures.

The present ninth grade experiences in general mathematics brought the Workshop Group face to face with a difficult problem. In studying the general mathematics program over the last twenty-five years it was found that general mathematics had been introduced into the ninth grade as a substitute for algebra with the hope that: (1) the number of failures in algebra would be reduced; (2) general mathematics would provide materials within the interest and understanding of the poorer students; (3) general mathematics would be a course of special value to those pupils who would not take mathematics after the ninth grade.

It was also found that an alarmingly large number of teachers considered general mathematics in the ninth grade a failure. The most prominent reasons given for this situation were: (1) the unfavorable and rebellious attitude of many mathematics teachers and administrators toward the subject; (2) the over-socialization of the course to the point of crowding out much of the essential mathematical frame work; (3) the over-loading of the

course, due to its terminal nature, to the point that neither understanding nor mastery could possibly be obtained.

Out of the preceding ninth grade general mathematics study came the conviction, to the Workshop Group, that all high school pupils not enrolled in the traditional courses should be required to take both ninth and tenth grade functional mathematics. It was considered important also that functional mathematics courses for grades eleven and twelve be prepared and be made available for those pupils who might choose to elect them. The Group deemed such action necessary in order:

1. To remove the over-load from the present ninth grade course
2. To spread the basic concepts, principles, skills, and materials over a longer period of time
3. To provide spiral learning of the concepts and principles

To the question, just how will this proposed sequence of functional mathematics courses fit into the present secondary mathematics curriculum?, the Workshop Group answers with the following diagram:

		Math. 9	Math. 10	Math. 11	Math. 12
Math. 7	Math. 8				
Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Algebra I	Algebra II	Geometry	Trigonometry
					Solid Geometry

From the diagram it is noted that a choice of tracks is available at the beginning of the ninth grade. A choice has been available at this point for many years, but now, instead of having a choice of general mathematics which is a dead-end at the end of the ninth year, a through track of four years of functional mathematics may be chosen. This functional track will greatly reduce the hazards of poor guidance and wrong choices which have been so prevalent in our present curriculum. The hazards will be reduced because functional mathematics, while differing considerably from the traditional in its purposes, is mathematically sound in its content. Moreover, because of the two-year requirement, it is believed by the end of the tenth year there will be about the same mathematical competence of an individual no matter which track is followed.

One would not want to infer that the proposed program could function at its best in the absence of guidance. It is quite conceivable that some of our average students, as well as the honored bright ones, can profit most by taking algebra. It is equally conceivable that some of our very brightest pupils, as well as those not so bright, should by proper guidance and choice select the functional program. These functional mathematics courses have been built to arouse the interests and meet the needs of bright pupils just as much as they have been built for any other type of pupil. So the procedure no longer should be as it has been in the past, that is, to pick out the so-called bright pupils and place them in the algebra classes and herd all others into ninth grade general mathematics. It becomes evident therefore that the need for guidance, better guidance than most of us right now seem to know how to practice, is still a very important part of this proposed program. However, should guidance fail entirely and the student get on the wrong track the damage to his mathematical abilities will not be serious.

The Work shop Group has much confidence in believing that the proposed functional mathematics courses will greatly strengthen our secondary mathematics program and raise the level of individual mathematical competence.

A Cooperative Plan to Share Objective-Type Test Items

By Henry W. Syer
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Teachers have long been advised to build and use objective-type test items in their classroom tests in addition to the use of standardized tests which have been constructed and sold by outside authorities. Our own classroom tests reflect the particular topics which we, as teachers, and our students have found interesting and important for our particular class during a particular year. Purchased tests can never fully replace the class tests made and used by the teacher who has carried the class along through the day-by-day development of the subject. However, all who have tried know the time and energy which is required to formulate worthwhile test items in mathematics. Sometimes items which seemed good do not work out in practice at all. If we all had time the ideal procedure would be to use, analyze, revise, use, analyze and discard items in a growing file which would thus be constantly refined and improved. Few teachers have time to follow through such a procedure individually. The purpose of this report is to indicate a procedure which might facilitate the exchange of items concocted by individual teachers of secondary mathematics through the country so that these items could be used by others. The suggested plan is to establish a regular department in The Mathematics Teacher which will collect, classify and publish items supplied by teachers who have written and used them. There is no thought of standardizing the topics or procedure in the teaching of mathematics; the items will be displayed for use, but no teacher is urged to use them if they do not meet the objectives of a particular class. As time goes by this pool of items may contain many which test the same concepts, skills or other objectives. This is all to the good for the bigger the selection, the more interesting the shopping tour.

Eventually, the best items may be skimmed off and published as separate collections in booklets for teachers to purchase, but this will be possible only if the cooperative plan described appeals to teachers and a steady flow of items is maintained. The National Council for the Social Studies has published such booklets concerning American History, World History, Economics, American Government and Study Skills and has found that they were used and appreciated by teachers.

Many of the considerations which are important when a complete test in a subject is being examined are not possible or appropriate when individual items are being collected. The most important is probably reliability. It would be very difficult to attach a meaning to the reliability of an individual item out of the context of a completed test. The other items, the position of the item in the test, and other factors influence the reliability. Also, we are usually interested in the reliability of the whole test and thus we leave this consideration to each teacher who chooses items from our pool to construct classroom tests. Other considerations which would be important for a whole test which we cannot discuss include the following: ease of administration, scoring and interpretation; and time and supplies needed. Specifically, there is no use in trying to call an item an "aptitude," "achievement" or "diagnostic" measure, since the same item may be used in various ways; this again depends upon the purpose and form of the test as a whole.

The following classification and symbolism of items will look awkward and

forbidding at first glance. It becomes simpler with use, and is intended to compress a great deal of information about each item into a small space. It will thus become apparent that the only items to be included are those which have been used and analyzed to some small extent. This means a bit of work for each person who submits items, but the whole purpose of this cooperative plan is to standardize such work and to share it enough with others to justify the time.

TABLE I

Subject Matter of Items

Code Letter	Subject Matter
G	General mathematics
C	Computation in arithmetic
R	Reasoning in arithmetic
E	Elementary (first year) algebra
A	Intermediate (second year) algebra
P	Plane geometry
S	Solid geometry
T	Trigonometry
V	Advanced mathematics
B	Business arithmetic
H	Shop or allied mathematics

In Table I appears a classification for the items to show the major field in which the item will probably be used. The code letter will be the first part of a code designation attached to each item printed. After some thought it was decided to make the primary classification of items in terms of the subject matter since this is the most common way which teachers will be searching for items to use. This is not to be interpreted to mean that it encourages water-tight courses organized around these areas, but is a simple admission that years of usage have given meaning to the subjects listed and for purposes of *classification* of items they are very useful; for purposes of *teaching* they may not be so useful.

All the meanings are self-evident except possibly G, V, B and H. *General mathematics* will mean any item which tends to overlap several fields combining, for example, material from algebra and geometry in one item. There will also be found many of the informational items of social mathematics now used in grades seven through nine. It is not necessary that the item be used in a course entitled "general mathematics" in order to be placed in this category. *Advanced mathematics* is a useful category for topics which are appropriate to a senior mathematics course, but are not solid geometry or trigonometry. Topics from third-year algebra (permutations, determinants, etc), topics from the analytic geometry and calculus which some find appropriate for the secondary schools will be placed here. *Business arithmetic* includes the items designed for courses of that name, and are fairly specialized items. Many which could be used here will appear under "general mathematics" because of their broader usefulness. Of course, such a classification as this we are attempting is neither perfect nor inviolate; any teacher forming a test will look through all categories which may possibly be appropriate. *Shop mathematics*, similarly, consists of specialized items useful in those course taught in conjunction with mechanical and trade courses. Many of the items listed under arithmetic will also be very useful in these courses.

Table II lists the various forms in which items may be cast. In mathematics the types most commonly used are completion, multiple-choice, computation, and true-false items, but there is no reason why we

TABLE II

Forms of Objective-Type Test Items

Code Letter	Form
A	Analogies
C	Completion
D	Deductions
E	Essays
F	Forms corrected for errors
G	Matching
H	Homologs
I	Identification
M	Multiple-choice
N	Constructions
O	Omission by cross-outs
P	Reproduction from memory
R	Rearrangements
S	Computations
T	True-false

should not explore and utilize less common types if they prove applicable. *Analogies* are the types of items which resemble mathematical proportions using concepts instead of numbers. For example: "Polygon:perimeter=circle:?" Such questions require the formulation of a relationship and then the finding of a concept which has that relationship. *Completion* items require the filling in of one or more blanks with words, symbols or numbers. *Deductions* are items which list a body of informational facts, usually in paragraph form, and then either ask for deductions which could be made from the facts, or supply deductions to be evaluated as possible or not possible. More subtle forms have more possibilities: absolutely deductible, possibly deductible, unable to determine whether deductible, possibly not deductible, absolutely not deductible, or some similar choices. *Essay* questions may be keyed so that they can be corrected in an objective manner, but this is neither a common nor a very satisfactory practice. *Forms corrected for errors* are statements which require the location and correction of errors within the statements; some, of course, in the set may be completely correct. An example might be, "A cube has 6 faces, 12 vertices, 8 edges, 4 diagonals and 16 diagonals of faces." *Matching* items consist of two columns of concepts, usually unequal in number, which are matched by association of meaning. *Homologs* may be association tests, or synonym-antonym items. In mathematics we might have one such as the following: "Give the word which means the opposite of each of the following: negative, rational, parallel, convex, and infinite." *Identification* items require classification of material given either by means of a list given, or from some classification retained in the memory. We might have in mathematics: "Which of the following could be regular polygons: square, isosceles triangle, parallelogram, circle, equilateral triangle, rhombus, right triangle, pentagon, trapezoid, and tetrahedron." *Multiple-choice* items are familiar. Since it is awkward having different numbers of choices for different multiple choice items within the same test, we will establish a ground rule that all multiple-choice items submitted will have exactly four choices. *Constructions* have the usual meaning of plane geometry, ruler and compass constructions, but they must be cast in the form of objective questions, with definite directions to the student and definite instructions to the person who will grade them. *Omission by cross-outs* would contain incorrect or redundant words or phrases which may be crossed out leaving the remainder correct. For example: "Cross out the words which make the statement incorrect: In geometry we are careful to define the following words: circle, diagonal, point, hypotenuse, parallel, ratio, triangle, and polygon." Or, for redundancies: "Cross out any redundant portions of this definition: The logarithm of a number n which is used to multiply and divide numbers is the exponent, usually written above and to the right of a number, which another number a, called the base, and usually taken as 10, or e (an irrational number), is raised to, to give the first number n." *Reproductions from memory* are seldom needed in mathematics, but might include the derivation of the quadratic formula or reproduction of complicated construction or similar facts. *Rearrangements* consist of a set of facts or ideas which have a natural, logical order, but are not in that order and must be rearranged. *Computations* should contain just one clear answer in order to beome objective-type items. *True-false* items are so familar that they need no discussion.

In Table III we find the classification of the types of materials which can be tested in the items. Since it is probably desirable to have only one clear-cut reason for each objective-type item, there should be no question of overlapping of objectives. It must not be thought that undefined words, definitions, unproved statements, and proved statements are reserved for the use of geometry courses; and rules,

formulas and skills, the special province of algebra. A moment's reflection will show that they all permeate all of mathematics. There is no doubt that the teaching as well as testing of applications, attitudes, appreciations, judgments and discriminations are deficient in the usual course in mathematics. They are included on the list for completeness, and to encourage the formulation of items for their evaluation.

TABLE III

Objective to be Tested by Item

Code Letter	Material
	Facts
U	Undefined words
D	Definitions
P	Unproved statements (postulates)
T	Proved statements (theorems)
R	Rules
F	Formulas
S	Skills
A	Applications
Q	Attitudes
N	Appreciations
J	Judgments and discriminations

So that teachers who wish to choose items from our stock may know what performance to expect from the item there should be some pertinent data attached to each. These should be supplied with the items by the teachers who send in contributions. Table IV tabulates the information needed. Originally, each item was doubtless part of a test constructed, administered, corrected and analyzed by a classroom teacher. The first bit of information needed is the total number of students who answered the item on the test. This should be distinguished from the number who took the test, and all who did not answer the item should be disregarded. In order to collect data on enough cases only items which have been answered by 100 or more students should be analyzed and submitted for publication. This should not be difficult, for it represents about three or four classes taking most tests. Next, we need the number of students who answered the item correctly. In order to describe the type of student who is represented by this group there need be three facts given: the age, grade and I.Q. of the group. This will enable other teachers to determine whether they wish to use the item with their groups. The validity of the particular item will be described in terms of the achievement of the bottom and top quarter of the whole group taking the test. By reporting the per cent (0 to 100%) of those in each quarter who answered the item correctly a fairly accurate indication can be given. Note that this must be the lower and upper quarter of the group who answered that particular item (correctly or incorrectly) and does not include those who omitted the item. This makes it more difficult to compute, than if it were the upper and lower quarters of the group taking the test, for many items may have been omitted for lack of time rather than ignorance. The computation of these figures will be greatly simplified if the students taking the test are encouraged to answer each item,—"guessing" they call it—and time enough is allowed for this to be done.

TABLE IV

Information Needed on Each Item

Code Letter	Information
N	Number of students answering item.
R	Number of students answering item correctly.
	Type of students answering item
A	Mean age and standard deviation of age
G	Mean grade and standard deviation
I	Mean I.Q. and standard deviation
	Validity information
Vb	The per cent of those in the lowest quarter (on the basis of the whole test) who answered this item correctly.
Vt	The per cent of those in the highest quarter (on the basis of the whole test) who answered this item correctly.

It may now be wise to give three examples of how individual items would be written up using the system described above. The items are new, and the data are fictitious.

AMD.0 [N—106, R—56, A—16.4 (±1.1), G—11.0 (±0.0), I—113 (±6); Vb—20, Vt—92]

Ans: (d)

$16^{-1/2}$ equals (a) $\frac{1}{8}$ (b) -8 (c) 4 (d) $\frac{1}{4}$

"AMD" means (from Tables I, II and III) that the item concerns intermediate algebra, that it is a multiple-choice item, and it is testing definitions. The number after the point will be a serial number telling how many of that type have been published. (Since this is a fictitious item, the number 0 is used.) All the information within the square brackets is identified in Table IV. It will be noticed that years and grades (and their standard deviations) are given to the nearest tenth, while I.Q. is given to the nearest whole number. These would of course be found by computing one more digit and rounding back (five and above would raise the digit). The validity per cents are given to the nearest per cent only. After the square brackets will be found the answer to the item. Here are two more sample items:

TRS.0 [N—221, R—144, A—18.2 (±0.3), G—11.8 (±0.1), I—121 (±2); Vb—40, Vt—90]

Ans: b e a c d.

Arrange in order of magnitude with the smallest value first:

(a) $\log_{10} 20$ (b) $\sin 25°$ (c) $\log_e 10$ (d) $(\sin 10°)^{-1}$ (e) $\sin^{-1/2}$ (in radians)

It will be noticed here that separate answers are identified by letters rather than numbers since there is less danger of confusing them with the answer itself.

PCN.0 [N—117, R—94, A—16.5 (±2.4), G—10.5 (±1.1), I—130 (±15); Vb—65, Vt—98]

Ans: Rectangle.

The most pleasing type of quadrilateral to use in art is the————.

There is the plan. If enough people are interested to make it a going proposition, the pool of test items should prove to be a tremendous help to teachers. It will not work if every one sits back and waits for others to supply him with items. Dig out the items you have, add to them, and let's get the department rolling. In order to utilize test items which may not yet have the information called for by Table III, it is possible to send in items without that information, if you wish to do so. However, this is not very desirable, since it precludes the possibility that they will be used in any later collection.

Items may be sent either to Mr. John H. Haynes, Acton High School, West Acton, Massachusetts, or the author of this article, who will act as joint editors.

The Signal Corps Posters

(*see pages 327–330*)

Editor's Note: On the center pages of this issue we are reproducing four of the twenty posters prepared and distributed in 1943 through the cooperative effort of the Signal Corps Schools of the Sixth Service Command, the Men's Mathematics Club of Chicago and the Women's Mathematics Club of Chicago and Vicinity. Unfortunately the supply was exhausted within a few months and the cost of preparing reprints in two colors at that time was prohibitive.

Shall we include the remaining sixteen posters in issues next fall? Our decision depends on the number of requests (a postcard will do) we receive from our readers, before June 15. Are we correct in estimating that 20% of our members will read this note and that 50% of those who read it may favor reprinting the other sixteen posters? Our question: How many responses will reach The Editor of The Mathematics Teacher, 212 Lunt Building, Northwestern University, Evanston, Ill., before June 15?

NOTES ON THE HISTORY OF MATHEMATICS

Edited By VERA SANFORD
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THE RULE OF FALSE POSITION

THE RULE of False Position, Single and Double, is a method of solving equations of the first degree in one unknown by guessing the answer and then adjusting the guess in terms of the conditions of the problem. As the name suggests, there might be one such guess or there might be two. The word *position* in this case is used in the sense of a "thing on which one takes a stand"—in its meaning here it is allied to the idea of letting or allowing, forming a tenet or proposition on which one expects to act. *Supposition* would be closer to our thinking.

The Rule of Single False is sometimes utilized implicitly by pupils who reason out a solution to problems involving simple equations rather than working with an unfamiliar algebraic notation. Such an instance occurred in an eighth grade class when the pupils were asked to find how the sum of $35.00 should be divided among three boys if it was understood that Tom should have twice as much as Dick and that Jim should have twice as much as Tom. One solution was that if Dick were given one dollar, then Tom would have two and Jim four. But under those circumstances, the total amount would be seven dollars not 35. But since five times seven is 35, the correct division would be to give each boy five times as much as had been computed in the first place—Dick would have $5.00, Tom would have $10.00 and Jim would have $20.00. The sum would be $35.00 and the amounts would be in the proper proportion. An instance such as this makes the Rule of Single False of interest to teachers of elementary algebra.

The Rule of Double False is important from the historical point of view because it furnished one of the important uses of signed numbers and because it illustrates clearly the way in which an understanding of computation with signed numbers enables a person to replace several rules by a single one.

The Rule of Single False appears implicitly in the Ahmes Papyrus (*c.* 1650 B.C.). There is reason to suppose that the two rules were used by the Hindus. They certainly were used by the Arab mathematicians. They were prominent features of mathematical works in Europe from Fibonnacci's *Liber Abaci* (1202) to the arithmetics of the sixteenth century. As algebraic symbolism developed, the Rules of False disappeared from the more advanced texts. For example, the rules were used in the arithmetics of Cocker and Leybourne in the late seventeenth century, but not in John Ward's *Young Mathematicians' Guide* (1706). The rules are given in Bonnycastle's *Scholar's Guide to Arithmetic* (1780) but not in the same author's *Introduction to Algebra*, an indication that the author considered it an arithmetic rule, not an algebraic one. Nicolas Pike's *New and Complete System of Arithmetic for the Use of the Citizens of the United States* (1788) gives a clear explanation of the matter and puts the algebraic proof in footnotes.

At first glance, the Rule of False is uncanny. It is reported that Robert Recorde (*c.* 1542) astonished his friends by proposing difficult questions and finding the true result from the chance answers of "suche children or yedotes as happened to be in the place." Humphrey Baker explained the name of the rule in his *Well spring of Sciences* (1568) in these terms:

"The Rule of falsehood is so named not for that it teacheth any deceit or false-

hood, but that by fained numbers taken at all adventures, it teacheth to find out the true number that is demanded. And this (of all the vulgar rules which are in practice) is the most excellent. . . . Those questions which are done by false positions, have their operations in a manner like unto that of the rule of three: but only that in the rule of three we have three numbers known, and here in this Rule, we have but 1 number that cometh in use to work by."

Nicolas Pike defines the Rule of Single Position in these terms: "Single Position teaches to resolve those questions, whose results are proportional to their suppositions (i.e., their guessed or assumed values). Such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times.

"Rule 1. Take any number, and perform the same operations with it as are described to be performed in the question.

"Then say, As the sum of the errors is to the given sum; so is the supposed number, to the true one required. *Proof.* Add the several parts of the sum together, and if it agrees with the sum, it is right." Pike then gives an example. A school master, being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I should then have 435. Of what number did his school consist?

(See top of next column)

Suppose he had	80
As many	80
$\frac{3}{4}$ as many	60
$\frac{1}{2}$ as many	40
$\frac{1}{4}$ as many	20
$\frac{1}{8}$ as many	10
	290

```
As 290:435::80
            80
290) 3480 0(120 Answer
     29
     58
     58
      0
          120
          120
           90
           60
           30
           15
          435 Proof.
```

This should be compared with the solution given in the Ahmes Payprus of the question to find a number such that the sum of the number and its seventh part is 19. In this case, the number is assumed to be 7, then the number and its seventh part would be 8, but it should be 19. Accordingly as many times as 8 is multiplied to give 19, so many times 7 must be multiplied to give the required number.

Problems solved by the Rule of Single False are varied. A popular one stated that the head of a fish weighs $\frac{1}{3}$ of the whole fish, the tail $\frac{1}{4}$ and the body weighs 30 ounces. What is the weight of the fish?

The problem of the pipes filling a cistern could be solved by this scheme and so too could the problem of the men doing a piece of work.

The Rule of Double False is more involved. Baker puts it this way:

"The summe of this Rule of Two false positions is thus, when any question is proposed appertaining to this Rule, first you must imagine any number at your pleasure, which you shall name the first position, and with the same you shall worke in stead of the true number, as the question doth import: and if you see that you have missed of the true number that you do seeke: then is the last number of the worke, either too great or too little: the which number, you shall note with the signe of more or lesse, for that is the first error, in the which you have failed, the which signes of more and lesse, shall be noted with these figures X—, This figure X, betokeneth more: and this plaine line—

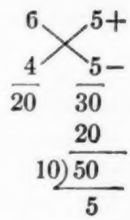
signifieth lesse: that it to say, the one signifieth too much, and the other too little: then you must begin againe, and take another number, which shall be the second position, and work by the question as before: if you have failed againe note the excesse or want, for that is the second error. Then shall you multiply the first position by the second error crosse-wise, and againe the second position by the first error (and this must alwaies be observed) and you must keepe the two products: then if the signs be both alike, that is to say, either both too much, or both too little, you shall abate the lesser product from the greater, and likewise you shall substract (*sic.*) the lesser error from the greater, and by the remaine of those errors, you shall divide the residue of the products, the quotient shall be the true number that you seeke. But if the signs be unlike, that is to say, the one too much and the other too little, then you shall adde those products together, and likewise you must adde both the errors together, and by the Sum of those errors, divide the totall sum of both the products; the quotient shall be the true number that you seeke."

Leybourne states that the Rule of False Position (and he restricts himself to the Rule of Double False) is suited to questions "which are not presently fit for the Golden Rule." Pike is more explicit—

"Those questions, in which the results are not proportional to their positions, (i.e. the guesses) belong to this rule: Such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required." Pike's illustration will help to clarify the rule quoted from Baker.

"A lady bought damask for a gown at 8 *s* per yard, and, lining for it at 3 *s* per yard: the gown and lining contained 15 yards, and the price of the whole was 3 *l.* 10 *s*; How many yards were there of each?"

Pike supposes first that there were 6 yards of damask, value 48 *s*. Then there would have been 9 yards of lining with value 27 *s* but this adds to 75 *s* which is 5 *s* too much. Then if there had been 4 yards of damask value 32 *s* and 11 yards of lining value 33 *s*. the sum would have been 65 *s* which is 5 *s* too little. Then he places the guesses at the left and the errors at the right as is indicated here:



He multiplies on the cross lines and adds getting 50. The sum of the errors is 10. Dividing 50 by 10 he reaches the conclusion that there were 5 yards of damask and 10 of lining.

The problem of deciding how to adjust the guesses and errors led to a number of memoriter devices. Cuthbert Tonstall (1522) went to his friend Sir Thomas More who wrote a stanza for him. He says "Et quo facilius has praeceptiones studiosi memoria tenerent, Morus, rogatu nostro, redegit eas in hoc carmen

> "A plure deme plusculum,
> Minus minori subtrahe:
> Pluri minus coniungito,
> Atque ad minus plus adiice."

Cocker in 1677 put it this way:

> "When Errors are of unlike Kinds
> Addition doth ensue
> But if alike, Subtraction finds
> Dividing work for you."

In brief, the cross products of guesses and errors are found and their difference is divided by the difference between the guesses. Pike's explanation of this occurs in a footnote to which earlier reference has been made. If a and b are the guesses and r and s the errors, N the given number and A and B the numbers that are produced by the guesses a and b, and x the number that is sought, then r is the difference between N and A; s is the difference between N and B, then $r:s::x-$

$a:x-b$ from which x is equal to $rb-sa/r-s$ "and if r and s be both negative, the Theorem is the same, and if r or s be negative x will be equal to $rb+sa/r+s$."

The operation of the Rule of Double False seems to have suggested recreational problems in which the results of two guesses were given. For an example of this there is a problem given by Calandri (1491).

"A school master has so many scholars that if each pays 8 pence he will lack 10 soldi of paying his rent, but if each pays 10 pence, he will have 20 soldi too much. How many scholars were there and what was the rent?" Clavius (1608) had a somewhat similar case in a problem which he attributed to Euclid,—If a mule takes one measure of wine from an ass's load, the mule will have double the ass's burden, but if instead the mule gives one measure from his load to the ass their burdens will be the same. How much was each carrying?

The need of indicating whether the errors were in excess or defect was the occasion for the introduction of the signs for positive and negative numbers into arithmetics printed in the sixteenth century. (It will be noted from the example given from Humphrey Baker that he used an X instead of the minus sign to which we are accustomed.) There is reason to believe that Widman who was the first to use the signs + and − in print introduced them into his arithmetic because he wished to have his students familiar with them for their use in the application of this rule. Humphrey Baker could claim that the Rule of False was the most excellent of all the common Rules, but that was in 1566. Today the rules are curios made obsolete by algebraic symbolism.

MATHEMATICAL MISCELLANEA

Edited by Phillip S. Jones
University of Michigan, Ann Arbor, Michigan

26. *More About Nedians*

Norman Anning of the University of Michigan generalizes as follows the concept of *nedian* as defined by John Satterly in The Mathematics Teacher for January, 1951 (vol. XLIV, pp. 46–48).

Assume that the points D,E,F on the sides BC, CA, AB of triangle ABC (Fig. 1) divide these sides cyclically so that the segments of each side are in the ratio $p:q$.

It then is not hard to show (he says) that

$$\frac{\Delta XYZ}{\Delta ABC}=\frac{(p-q)^2}{p^2+pq+q^2} \tag{1}$$

$$\frac{\Delta DEF}{\Delta ABC}=\frac{p^2-pq+q^2}{(p+q)^2} \tag{2}$$

$$\frac{\Delta XYZ}{\Delta DEF}=\frac{(p^2-q^2)^2}{p^4+p^2q^2+q^4}. \tag{3}$$

Of course (3) follows from (1) and (2). When (3) is compared with (1) one observes it to be merely (1) with p replaced by p^2 and q replaced by q^2. This leads to the conclusion that YZ cuts EF, ZX cuts FD, XY cuts DE in the ratio $p^2:q^2$ and thus suggests that within ABC there is a readily constructable hierarchy of triangles having AD, BE, and CF as nedians and ΔXYZ as a limit.

Professor Anning's formula (1) specializes to Professor Satterly's formula (3) for the area of the nedian triangle. Let $p=1$ and $q=N-1$, then $BD=(1/N)BC$ and

$$\frac{\Delta XYZ}{\Delta ABC}=\frac{(1-N+1)^2}{1+N-1+(N-1)^2}$$

or

$$\Delta XYZ=\frac{(N-2)^2}{N^2-N+1}\cdot\Delta ABC.$$

Please note that this also constitutes a correction of a sign error in the denominator of the Satterly formula as previously printed.

A second and voluminous note came from MERTON T. GOODRICH of Keene Teachers College, Keene, New Hampshire. Professor Goodrich had been working independently on essentially the same idea for some time prior to the appearance of Professor Satterly's note. Goodrich had invented the term *redian* drawing the *r* from *ratio*. He has extended the concept from triangles to parallelograms, and polygons in general. He states that he has obtained such theorems as that the central *redian* figure of any polygon is the same kind as the original polygon, and that in any triangle, parallelogram, or regular polygon what he terms the *redian* triangles, one for each vertex, are all equal.

For our Figure 1 the *redian* triangles are:

$$\Delta AFY = \Delta BDZ = \Delta CEX = \frac{1}{N(N^2 - N + 1)} \cdot \Delta ABC.$$

Further for the quadrilaterals:

$$FBZY = DCXZ = EAYX = \frac{N^2 - N - 1}{N(N^2 - N + 1)} \cdot \Delta ABC.$$

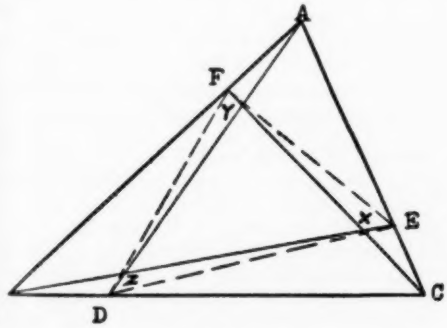


FIG. 1

Interestingly, these same formulas were also relayed to us from Professor Petrie in a letter from John Satterly.

We hope to see more of Professor Goodrich's interesting work in print later on. Perhaps other readers, too, have further results which they will send in.

27. Geometric Progressions—Two Notes

KARL S. KALMAN of Abraham Lincoln High School, Philadelphia, Pennsylvania writes that in addition to the usual more rigorous derivation of the formula for the sum of an infinite geometric progression, $S = a/1 - r$, using the limit concept, he finds the following presentation adds interest and conviction for his students.

Let $S = a + ar + ar^2 + ar^3 + \cdots$ where a is the first term, r is the constant ratio, $r < 1$, S is the "sum" (which it is assumed exists) then

$$S = a + r(a + ar^2 + ar^3 + \cdots)$$

$$S = a + rS$$

$$S = \frac{a}{1 - r} \cdot$$

To add further insight into and "meaning" for this theorem Mr. Kalman uses the device of specialization. He calls the attention of his students to the case $a = 1$, when $S = (1/1 - r) = 1 + r + r^2 + r^3 + \cdots$. It is particularly evident in this case that if $r = 1$ then the limit of the series does not exist. This is beautifully consistent with the fact that the fraction $1/1 - r$ is not defined for $r = 1$. Further, if $r > 1$, the formula clearly gives a negative value for S while the series is just as clearly positive. With this heuristic treatment students will not only recall longer the condition $r < 1$, but will also feel it to be more intuitively sensible as well as mathematically necessary.

A. R. JERBERT of the University of Washington, Seattle, Washington contributes the following note which he titles *Clock Problems—Achilles and the Tortoise:*

It is no longer the fashion to include clock problems in beginning algebra texts. The reason can't be that they are

unpractical because that is the case with nearly all the problem material which is available to a first course in algebra. In the matter of difficulty, too, they compare favorably with work problems, time-rate problems, and a host of others.

A typical clock problem is to determine "at what time between 9 and 10 o'clock are the hands of a clock opposite to each other?"* Evidently the minute hand must *gain* 15 minute spaces. Thus,

let x = number of spaces traversed by the minute hand,

and $x/12$ = number of spaces traversed by the hour hand, so that

$$x - x/12 = 15$$

whence,

$$x = 15(12/11) = 16\tfrac{4}{11}\cdot$$

For the student who is familiar with the concept and formula it is just as expeditious to "sum" an infinite geometric progression. Thus we can argue that the minute hand must cover 15 spaces (at least). However, while this is going on, the hour hand moves up 15/12 spaces. The minute hand must therefore cover this many spaces in addition. Continuing in this manner, the minute hand must cover in all

$$15 + 15/12 + 15/12^2 + \cdots,$$

spaces. Since $r = (1/12) < 1$,

$$S = 15/(1 - 1/12) = 15\cdot(12/11), = 16\tfrac{4}{11}$$

as before.

If we substitute Achilles and the tortoise for minute and hour hand respectively, we have essentially the classical problem which Zeno proposed as a paradox to confound rival Greek philosophers. Rephrase it and we have: "Assuming that Achilles travels 10 times as fast as the tortoise, how long will it require for him to overtake the tortoise if the latter has a 100 yard start?"

Analysis and solution: While Achilles is running the 100 yards which separates him from the tortoise, the latter moves ahead another 10 yards. In the time that it takes for Achilles to cover the additional distance, the tortoise moves forward another yard. Thus Achilles must run successively,

$$100,\ 10,\ .1,\ .01,\ \cdots,$$

yards. Since this series runs on indefinitely, it would (and did) seem evident that Achilles would never catch up with the tortoise. As with the clock problem, however, we have merely to employ our formula to obtain $(10/9)\cdot(100)$ as the total number of yards which Achilles must run to overtake the tortoise.†

28. Card Sorting and the Binary System‡

A widely used application of the binary number system occurs in connection with punched card systems of the McBee Keysort type (Fig. 2). The application arises, of course, from the fact that there are just two choices available for a given

* The writer recalls a "perennial student" in a beginning algebra class, who was essaying the course for the third or fourth time. When clock problems came around, this man (he was 20 or more) pulled out a gold watch and solved them empirically. Unfortunately, the expensive watch failed to "turn up" the odd elevenths which are yielded by a little pencil work.

† *Editor's Note:* Gregory St. Vincent in his *Opis Geometricum quadriturae circuli,* (Antwerp: 1647) was the first to apply this procedure to the "Achilles" problem stated by Zeno circa 450 B.C. according to Florian Cajori, *A History of Mathematics* (New York: 1938), p. 182.

‡ Historical and recreational applications of binary notation have been discussed by Vera Sanford in "Notes on the History of Mathematics," The Mathematics Teacher Vol. XLIV (January, 1951), pp. 29–30. A treatment of uses of binary notation in electronic computers may be found in L. N. Ridenour's article, "Mechanical Brains," *Fortune* Vol. XXXIX (May, 1949), pp. 109–118. In the appendix of this article Ridenour applies the binary notation to the mathematical theory of the game of Nim. The "Russian peasant" multiplication procedure, the theory of which rests upon the binary system, is described in Oystein Ore, *Number Theory and Its History,* (McGraw-Hill Book Co., Inc., 1948), pp. 38–39 and Sir T. L. Heath, *A Manual of Greek Mathematics,* (Oxford, 1931), p. 29.

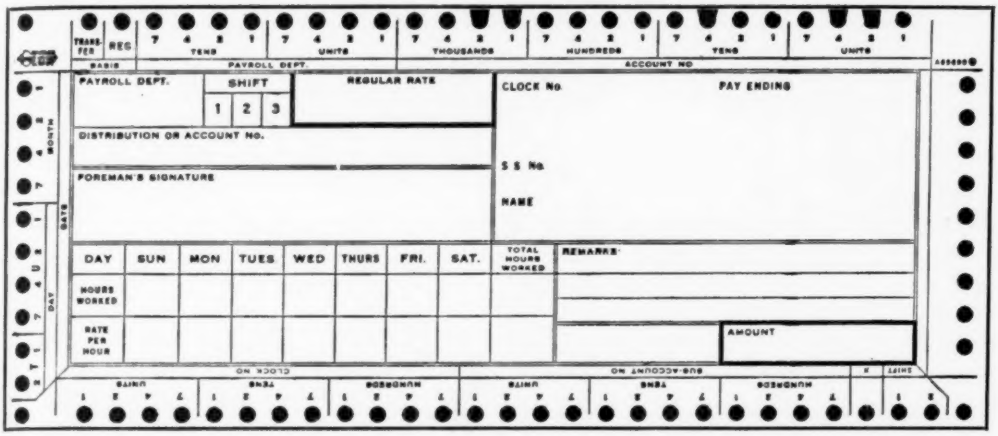


FIG. 2. A McBee Keysort Card.* Manufactured by the McBee Company, Athens, Ohio.

position on the card—there may or may not be a hole punched there.

By the use of what is called a "sequence code," the cards in a pack are coded serially. The serial number for each card is designated in binary notation, with two modifications: (1) the decimal *place* system is retained, there being portions of the card set aside for the units, the tens, the hundreds, etc., places of the serial number with the binary system being used merely to specify the digits from 0 through 9 which should appear in that place; and (2) the binary system itself is altered so that instead of the first four places (right to left) representing $2^0=1$, $2^1=2$, $2^2=4$, $2^3=8$ respectively, the digits used for coding are 1, 2, 4, and 7.

The 7 is used in place of the 8 because this substitution makes it possible to specify any of the numbers from 0 to 9 by not more than two code numbers. Standard binary notation for these numbers would have been:

0— 0
1— 1
2— 10
3— 11
4— 100
5— 101
6— 110
7— 111
8—1000
9—1001

In this list, only the number 7 requires three non-zero digits (and hence three punches on the card) for its representation. With the altered notation (the fourth place representing 7 rather than 8), the digits 7, 8, and 9 become respectively 1000, 1001, and 1010. This departure from strict binary notation, it may be noted, results only in the loss of the ability to designate the number 15, an unimportant defect here since we need to represent only the digits 0 through 9.

In the Keysort system, the cards as furnished by the manufacturer have holes punched around the borders. Information is written on the face of the card, and a hole on the border is designated to refer to each item of information. When a card contains a certain item, that fact is indicated by punching out a V-shaped notch at the position of the corresponding hole, so that the hole is replaced by a notch in the edge of the card. For example, Figure 2 pictures a card notched to correspond to the number 3046. When it is desired to select, out of a pack of cards, those which contain a particular item, a long needle is run through the pack at the position of the hole for that item. The needle is then lifted, and those cards

* Reprinted by permission from the manufacturer.

which have been punched at that position may be made to fall out, while the rest of the cards remain on the needle. This is the basic sorting procedure, and the procedures for performing the more complicated types of analyses are merely extensions of it, involving somewhat more elaborate equipment.

As the cards are used for various purposes, they are likely to get out of their serial order. The restoration of this order is easily accomplished, requiring only 4 sorts for each digit of decimal notation. For example, suppose a pack of the cards are in jumbled order, and it is desired to place them in serial order. The sorting is first performed in the units portion of the card. The units digits will have been coded as follows:

0—no punch
1—1
2—2
3—2, 1
4—4
5—4, 1
6—4, 2
7—7
8—7, 1
9—7, 2.

All cards coded with a 1 are first sorted out and put at the back of the pack. This will divide the pack into two parts. In the first part will be the cards with units digits 0, 2, 4, 6, 7, or 9; and in the second, those with units digits 1, 3, 5, or 8. This situation may be indicated:

0, 2, 4, 6, 7, 9—1, 3, 5, 8,

and it should be kept in mind that the cards are in jumbled order in each group. It is not necessary, during the sorting process, to keep the groups separate.

A sort is now made by the digit 2, and the cards sorted out placed at the back of the pack. This will give the following grouping:

0, 4, 7—1, 5, 8—2, 6, 9—3

and it may be observed that all cards coded 3 are together.

Sorting by 4 gives:

0, 7—1, 8—2,9—3—4—5—6.

The 4's and 5's and 6's are all together because each came from a different group out of the preceding sort.

Finally, sorting by 7 will result in all the cards being in proper order according to their units digits:

0—1—2—3—4—5—6—7—8—9.

The same sorting procedure is next followed using the code numbers for the tens digits. It may easily be verified that at the end of the process the cards will all be in order according to their last two digits. The process of four sorts per digit is then repeated as many times as there are digits in use on any of the cards.

A parlor-trick type of illustration of this sorting technique may be performed with a deck of ordinary playing cards. Assigning the values 11, 12, 13 to the Jack, Queen and King, and using the code digits 1, 2, 4, 8 makes it possible to arrange the cards in order of value in 4 sorts, sorting the cards into two piles each time.

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University of Michigan
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29. Sorting Jumbled Cards into Numerical Sequence

The previous note recalled to the editor an experience of the bankrupt days of the 1929's when he was employed by a receiver to sort into a numerical sequence a large number of invoices which had been filed alphabetically. In his youthful innocence he started to sort into piles by hundred thousands, then these into subpiles, etc. Working space was soon exhausted and confusion reigned. An efficient procedure would have been to sort into ten piles by the units digit, assemble the piles in order into a single bunch and re-sort into ten piles by the tens digit, repeat this process until all the places in use had served as a basis for a sort. At this time the invoices would then be in proper serial order.

AIDS TO TEACHING

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Readers of this department are invited to send information about new or recent "Aids to Teaching" either to Dr. Syer or to Dr. Johnson. Information about items which have not been reviewed to date is especially welcome but suggestions on other available items are needed also. If possible, include name and address of distributor or publisher, cost, grade level at which material might be used, and any comments as to the effectiveness of this material when used in the classroom.

The material reviewed is checked with the producer to determine whether it will be available the month when the review appears; sometimes conditions change before publication, however.

Many companies prefer to send one free copy of any material to a teacher and not to supply quantity for a whole class. Since this depends upon the number of requests made, it would be well to ask whether a quantity is available.

BOOKLETS

B.59—The Outlook for Women in Mathematics and Statistics

Superintendent of Documents, U. S. Government Printing Office, Washington 25, D.C.

Booklet; 6″×9″, 21 pages; U. S. Department of Labor, Women's Bureau, Bulletin Number 223–4; $.10.

Description: This booklet is primarily concerned with the changes and trends in the kind of work and the supply and demand for women in mathematics and statistics. It includes a large amount of data showing the number of persons trained and working in these fields. It describes earnings and opportunities for advancement. On the basis of the new positions resulting from our technological advances, the booklet attempts to forsee the needs and opportunities for mathematicians and statisticians in the future.

Appraisal: The information for this bulletin was culled from 800 books, articles, or pamphlets as well as from such primary sources as scientific organizations, employers and trainers of women scientists, and men and women scientists themselves. Besides giving the teacher and personnel worker valuable information for the guidance of superior mathematics students, the description of the kinds of work performed by mathematicians, statisticians and actuaries will indicate the applications of mathematics in a variety of situations. It will give the mathematics teacher useful information to show that training in mathematics is becoming increasingly important.

B.60—Geometric Constructions

New York University Bookstore, 18 Washington Place, New York 3, N.Y.

Mimeographed booklet; $8\frac{1}{2}''\times 11''$; 59 pages; $.75.

Description: This booklet, written by Aaron Bakst, summarizes and organizes various methods of determining a solution of problems in construction. The methods are grouped under the following headings: Loci, parts of figures, similarity, algebraic method, motion, symmetry, inversion, perspective, and trigonometric method. There is a bibliography of 25 items at the end, four of them in English. A short discussion of other methods than straightedge and compasses is included at the end.

Appraisal: This is a splendid summary of the subject with many helpful examples worked out. It is far more complete than most secondary school pupils will need, and may even trouble some teachers with its completeness. Still, better to have it too complete than less complete. Without doubt more work could be done with con-

structions in classes if teachers could see the possibilities of their being considered miniature research projects, as the author of this booklet does. Unfortunately, many teachers and pupils do not have the "research mind," and the abstractness of mathematics as a field for learning about research, although theoretically ideal, does not appeal to them.

It has been noted that the types of construction problems discussed are often more difficult than we have time and interest to teach. On page four it is stated that the purpose is "to develop the system of geometric constructions along the model of a mathematical system." This development is not obvious in the rest of the booklet; nor are the fundamental constructions independent. In general however, this booklet will be interesting and valuable for teachers to read and should influence their teaching for the better.

B.61—Math Miracles

Wallace Lee's Magic Studio, P. O. Box 105, Durham, N. C.

Bound book; $5\frac{1}{2}'' \times 8\frac{1}{2}''$; 83 pages; $3.00.

Description: Here are mathematical tricks in the form of magical presentations. Most of them are familiar, but the selection is good. Enough hints are given of the way to present them so that an appealing performance can be given. Twenty topics are used; casting out nines, magic squares, and similar topics are mentioned.

Appraisal: To enliven mathematics a program of these tricks could be presented by pupils who have learned them. There is plenty of opportunity to learn mathematics by analyzing why the tricks work. Some are too difficult to analyze at the level where they are very interesting and acceptable as tricks, and could be enjoyed for their own sake at that level. After all, there is no reason why we must always pull to pieces everything mathematical; for some people the play with numbers is satisfying and beautiful without the understanding. Possibly the aesthetics of arithmetic deserve more attention.

The binding and illustrations are quite attractive. The hard cover should last for a long time.

CHARTS

C. 25—Charts for Arithmetic

F. A. Owen Publishing Company, Dansville, N. Y.

Ten charts; $10'' \times 13''$; $1.00.

Description: Since each of the ten sheets is printed on both sides, there are really twenty posters to display. They are printed on bright colored sheets of light cardboard and enclosed in a portfolio. The following subjects are covered: measures, a time chart, linear measure, Roman numerals, change for a dollar, kinds of subtraction, temperature, weight, liquid and dry measure, measurement by counting, decimals, fractions, interest problems, long division, and percentage. These are mostly useful in the intermediate grades.

Appraisal: The purpose seems to be both understanding and drill. The material is a heterogeneous mixture of tables of denominate numbers, problems to be solved, and graphical representations of fractions. All material could very easily be written on the blackboard, but is probably useful to have immediately at hand for display. Only five of the twenty contain anything other than numbers and words, and then only circles to represent fractional parts, or pieces of money. It is unfortunate that some enlivening illustrations could not be included. There is a need to supplement the drill with applications, or, better still, to achieve the drill through applications.

BULLETIN BOARD DISPLAYS

D. 1—Problem of the Week

Mr. Mel Lieberstein, Collinsville Township High School, Collinsville, Ill.

Mimeographed sheets, 45 in all; $8\frac{1}{2}'' \times 14''$; $2.50 per set, $6.50 for 3.

Description: This set of 22 problems is mimeographed on a large scale so that

they are all ready to mount on bulletin boards. There is one horizontal title sheet which serves as a caption. The other sheets are vertical and contain the problem on one sheet and the answer on another. There is room on the answer sheets to record the names of pupils who submitted the correct answers to the problem of the week before.

Appraisal: The idea is clever, the problems are quite appropriate for secondary school pupils, and the job of mimeographing is extremely well done. The line drawings which serve to illustrate some of the sheets are attractive and add a great deal to the interest in the display. By coloring some of the sheets by hand, even more interest may be aroused. The price charged is far too much for the material received; it should be sold at no more than $1.00. Then individual teachers could be supplied with sets to use. The best method now is to acquire one for a whole school system and circulate it for all teachers to use. By making other caption sheets by hand and sending the sheets from school to school, many teachers could use the set. Another way would be to look up similar problems in books on mathematical recreations and have sheets made in local classes. These could be mimeographed for all, or done by hand by individual pupils. If necessary to get the idea started, this set could be bought.

EQUIPMENT

E. 39—Aritho

Psychological Services, 4502 Stanford Street, Chevy Chase, Maryland

Game; $12'' \times 13\frac{1}{2}''$; $1.50.

Description: This game consists of fifteen answer cards (each $4'' \times 5\frac{1}{2}''$) with twenty-five numbers on each arranged in the form of a square. The five columns are headed $+$, $-$, $\div$, $\times$ and $\times$. There are six problem cards which are cut up into 390 problems in addition, subtraction, multiplication and division. There are also many small squares of cardboard to cover answers on the answer cards. The game is played like Bingo. A caller picks up a small problem card and reads the problem aloud. The answer must be covered in the correct column of the answer card which any player holds. Some one keeps track on a master sheet of the answers which are used to check. Only integers are used in problems and answers.

Appraisal: This seems to be capitalizing on a familiar and fascinating kind of competition. Its purpose, is, of course, a painless method of drilling on number combinations. There is no reason why a whole closet full of these games should not be owned by the schools and used constantly. Once the meanings of the operations are taught, the drill should follow up to the desired level of mastery.

The answers to addition range from 0 to 18; those in subtraction and division from 0 to 9; and those in multiplication include 0–10, 12, 14, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 49, 54, 56, 63, 64, 72, and 81. This gives some idea of the scope of combinations included. The fundamental combinations are the chief ones used. All the addition facts from $0+0$ to $9+9$ are used; all the subtraction facts from $0-0$ to $18-9$ are used; all the multiplication from 0×0 to 9×9; and all the division from $1\div1$ through $81\div9$ are used. There is some doubt about the advisability of arranging the problems with the second number under the first in every case. It would be wise to have both the horizontal and the vertical arrangements for addition, subtraction and multiplication. For division the horizontal arrangement is probably more familiar than the vertical which is used. Since only the caller sees the form, however, this is probably a minor point.

This game should provide many exciting minutes in the drill of number combinations.

E. 40—Criteria Quadrilateral

W. M. Welch Scientific Co., 1515 Sedgwick Street, Chicago, Ill.

Device; aluminum; Catalog No. 7520; $6.25 each—$3.25 each for 100 or more.

Description: This device consists of two aluminum strips which rotate about a point at which is mounted a circular protractor. On each member are two fixtures which can be moved relative to the pivotal point. The aluminum members represent the vertices of a quadrilateral. Each diagonal is graduated in centimeters reading outward from the point of intersection. An elastic thread stretched around the four fixtures (vertices) represents the quadrilateral. This device is designed to show how different diagonal relations determine the different types of quadrilaterals.

E. 41—Parallel Lines Device

W. M. Welch Scientific Co., 1515 Sedgwick Street, Chicago, Ill.

Device; $14\frac{1}{2}'' \times 12''$; aluminum; Catalog No. 7515; \$5.50 each—\$2.75 for 100.

Description: This instrument consists of two colored aluminum strips and a transversal having circular protractors mounted at the points of intersection. Each device is equipped with a snap-on fixture which holds the lines parallel. Thus, it can be used as a parallel rule for drawing parallel lines. Lines are etched on the strips to avoid considering the strip as a line. The lines can be rotated until they appear to be parallel and the relative sizes of the various pairs of angles can be read from the protractors; or, the sizes of the angles can be controlled and the relative positions of the lines can be visually checked. The theorems relating to alternate interior angles, corresponding angles, exterior angles, interior on the same side of the transversal, etc., can be investigated or verified. With the snap-on fixture in place, all the theorems relating to parallel lines cut by a transversal can be illustrated.

Appraisal of E. 40 and E. 41: These simple devices are similar to other Schacht Dynamic Geometry Instruments previously evaluated in this section. They are well constructed devices that should make it possible for the geometry teacher to illustrate relationships within a quadrilateral and also between parallel lines more dynamically than with drawings. Although the price for one or two seems high, they compare favorably with the cost of equipment for the science laboratory. In addition, the rapid drop in cost as additional sets are ordered, for example, orders for three to nine units ofthe parallel lines device cost \$4.00 each, should stimulate large orders so that they will be available for student manipulation. To be effective they should be used by the student to discover or verify relationships. From such an investigation only tentative statements of probable relationships can be made. After the theorems have been established, the devices provide concrete representations of them for review and emphasis.

E. 42—Quad

The Gangler-Gentry Co., Catonsville 28, Maryland

Game; $3'' \times 5'' \times 5''$; \$1.00 each, \$6.90 per dozen.

Description: This game consists of a plastic base and stem arising from it; on the stem are three square platforms marked into sixteen equal squares on each platform. The base is similarly marked. A cardboard box set into the base holds eighteen markers of each of three colors. The game can be played by two, three, four or six people. The side wins which can get four in a straight line first. There are seventy-six ways that this can be accomplished.

Appraisal: There is some amount of space-perception involved when a line of four counters, all on different platforms, is being arranged. This would serve as a bit of three-dimensional thinking in a plane geometry course, or as an introduction to lines in space at the beginning of solid geometry. Once the game is understood it might be well to replace it by a set of four tic-tac-toe blocks drawn side by side on a piece of paper or the blackboard. Each square would contain sixteen small squares and, by imagining them piled on top of each other, one can play

a similar game involving even more space-perception.

This game is well made and will prove of fair durability. Of course, the small markers will be lost, but can be replaced. All thoughts of teaching mathematics aside, this is an interesting and exciting game.

MODELS

M. 23—Mek-N-Ettes

The Judy Company, 310 North Second Street, Minneapolis, Minn.

Toy; Mounting board, plastic gears, bars, discs, etc; $4.50 to schools.

Description: This educational toy has the necessary gears, bars, and pins to show many modifications of motions. Two gear sizes, 1-inch and 3-inch, can be used to build gear ratios up to 1 to 243. Various gear trains can be built to show changes in speed as well as direction of motion. Elliptical gears and a straight toothed rack, as well as cranks and levers can be used to illustrate other methods of mechanical power transmission. The slotted bars can be used as connecting rods or lever arms to set up linkages which convert rotating motion to motion in a straight line. A manual of instruction gives directions and photographs of several simple mechanisms.

Appraisal: This mechanical toy will be useful for the science and mathematics teacher to show different types of machines and the formulas that apply. It will give the teacher simple, inexpensive apparatus for laboratory work in which the student can discover relationships and formulas. These principles can then be applied to show the development and operation of a great variety of every day machines. The board on which the devices are built is used like a graph in that the positions for attachment are located by coordinates. The most appropriate grade level for its use is probably junior high school, although it can frequently be used to illustrate more advanced topics such as vectors.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by Emil J. Berger
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

A Simple Trisection Device

The trisection device illustrated in Fig. 1 is an interesting application of several important geometric theorems.[1] One of these is that arcs of a circle cut off by parallel lines are equal. Proof of the device for a problem in application will show how this theorem is employed.

A workable device may be built with four narrow strips of either wood or cardboard. For a blackboard demonstration device the dimensions of these strips should be as follows: AE and DB, each 12″ long; CS and CR, each 18″ long. To assemble the device find the midpoints of AE and DB, and locate A and B on CR and CS respectively so that $CA = CB = 6''$. It can readily be seen that quadrilateral $ACBO$ is a rhombus with sides of 6″. If wood is used for the strips, the movable joints, A, B, C, and O may be secured with small brads. If cardboard strips are used, holes should be punched at the movable

[1] Plans for construction of the trisection device described in this article were developed in the Mathematics Laboratory, Monroe High School, St. Paul, Minnesota. It is a modification of a similar device which appears in the *Eighteenth Yearbook* of the National Council of Teachers of Mathematics: Yates, Robert C., "Trisection," p. 151.

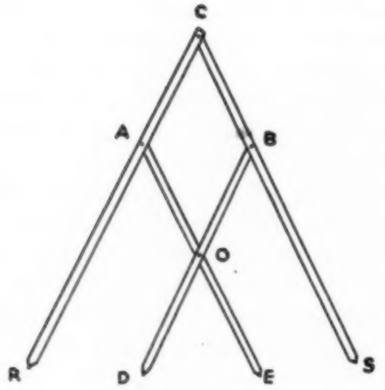


Fig. 1

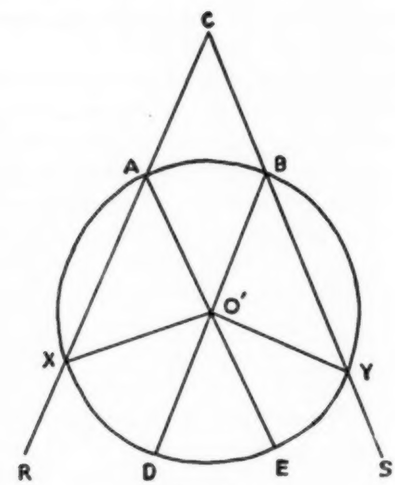


Fig. 2

joints, and paper rivets inserted.

To trisect any angle $XO'Y$ with this device, draw a circle of radius 6″ with its center at vertex O'. (See Fig. 2.) Sides $O'X$ and $O'Y$ should be extended until they intersect the circle. Place the device on the angle so that O falls on O', CR passes through X and CS through Y. Since the circle O' is a twelve-inch circle by construction, points $A,B,D,$ and E all lie on the circle. The following proof shows that trisection of angle $XO'Y$ is immediate.

Proof

(1) Angle $AO'B$ = Angle $DO'E$.	(1) Vertical angles are equal.
(2) Arc AB = Arc DE.	(2) Equal central angles have equal arcs.
(3) Arc AB = Arc XD.	(3) Arcs of a circle cut off by parallel lines are equal.
(4) Arc AB = Arc EY.	(4) Reason (3).
(5) Angle $XO'D$ = Angle $DO'E$ = Angle $EO'Y$.	(5) Equal arcs have equal central angles.

A Multiple Purpose Device

The device to be described in the following paragraphs is a favorite of this contributor because of its simplicity and usefulness. It may be used to help clarify the differences between the medians and altitudes of a triangle, to illustrate several types of loci, to show how area and perimeter are related, and to demonstrate facts about parallel lines.

The materials needed for construction include four narrow extensible curtain rods, four large letters, four metal paper clamps, one screw eye, a quantity of elastic, colored cord with alternate half-inch bands of red and white, and two wooden uprights $\frac{3}{8}'' \times 1'' \times 4'$ (or long enough to fit firmly inside the frame of the blackboard).

To assemble the device fasten the curtain rods R_1, R_2, R_3, and R_4 to the uprights in such a way that all of them will be parallel to the base of the blackboard. The distances between them should be in the ratio 2:3:3. (See Fig. 3.) Next glue the letters A, B, C, and D to the four metal clamps, and clamp those bearing letters A, D, and C to rod R_3 so that AD will be equal to DC. Finally place the clamp bearing the letter B on rod R_2.

By running an elastic cord through the holes in the handles of the clamps, a triangle may be formed with the vertex angle at B. By joining B and D with an elastic cord, a median of the triangle is outlined. If the screw eye is suspended from B by means of colored cord then it can be made to serve the purpose of a plumb bob and the cord itself will outline an altitude of the triangle.

The difference between a median and an altitude of a triangle drawn from the same vertex may be demonstrated by sliding the vertex clamp along rod R_2

from one end to the other. As the triangle approaches the isosceles in shape, the altitude approaches the median. By starting with the isosceles triangle one can show the altitude leaving the median and swinging farther and farther away until it is entirely outside the triangle.

The difference between the median to side AC and the bisector of angle B can be clarified by sliding the vertex of the triangle to an extreme position.

The relationship between the altitude and the base of the different kinds of triangles can also be demonstrated with this device. If it seems desirable the median may be removed for this exercise. In the isosceles triangle the altitude from the vertex B bisects the base; in the general acute triangle it falls inside the triangle; in the right triangle it coincides with one of the legs, and in the obtuse triangle it falls outside the base. The class can see all the possible situations in rapid succession simply by sliding the vertex clamp along R_2 and allowing the triangle to change shape as desired.

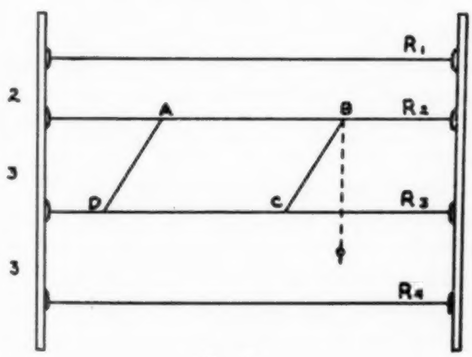


FIG. 3.

The contrast between the perimeter and area of a triangle can be illustrated rather nicely by moving the vertex clamp from the left until an isosceles triangle is formed, and then on to the extreme right. The area remains the same for all positions of the vertex on R_2 but the perimeter changes perceptibly.

Remove the elastic joining D and B, and place clamp A on rod R_2. Then connect D and A with elastic; the resulting figure is a trapezoid. (See Fig. 4.) By spacing the clamps so that $AB=DC$ a parallelogram is formed. The shape of this parallelogram may easily be changed by altering the positions of A and B. Thus it is possible to show that the altitude from B to the base DC may fall inside, on, or outside the figure. Following are some of the theorems that can be illustrated with the "new" device.

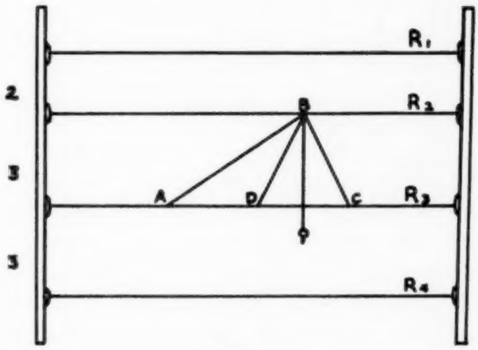


FIG. 4.

(1) The area of a rectangle is equal to the product of its dimensions.
(2) The area of a parallelogram is equal to the product of its base and altitude.
(3) The areas of all parallelograms with equal bases and altitudes are equal.

In connection with the third theorem it should be noted that the device is helpful in showing that the area of a parallelogram does not depend on its perimeter, and also that of all equal parallelograms with equal bases the rectangle has the least perimeter.

The device can also be used to demonstrate several important locus theorems:

(1) The locus of vertices of all equal triangles with a common base is two parallel lines, each parallel to the base, with one on either side of it, and at the distance of the altitude from it. (Use R_3 as the base.)
(2) The locus of points at a given distance from a given line is two lines parallel to the given line, one on either side of it, and at the given distance from the line. (Use R_3 as the given line.)
(3) The locus of points equidistant from two parallel lines is a line parallel to the two given lines and midway between them.

Remove the clamps A, B, C, and D, and four parallel bars remain. Clamp a tailor's tape measure to R_2 and R_4. The result is a transversal which is divided into

two equal parts. Move the clamps to different positions on the same two lines and it can be seen that every transversal formed by the tape is divided into two equal parts. If the tape is clamped between R_1 and R_4, it is divided into segments whose ratio is 2:3:3

Louise B. Eddy (Mrs. John M.)
Chicago, Illinois

Fact Finders

The "fact finders" suggested in this article are intended for use with children of the primary grades. The two devices may be used simultaneously. The first—the simpler of the two—is a learning aid; the second is a demonstration teaching aid. Each pupil in the class should be supplied with one of the first type, but only the teacher needs to have a demonstration device. The combined use of the two devices should make possible a more meaningful presentation of the counting process and the number combinations.

For first grade pupils the learning aid consists of ten colored beads strung on a ten-inch wire which has a loop at each end. (See Fig. 5.) Coat hangers may be used as a source of supply for the wire.

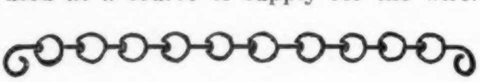


Fig. 5.

The demonstration device to be used with first graders is about 20 inches long, and the wire is mounted on a wooden frame with a shield at one end. The purpose of the shield is to hide beads that are not in use. (See Fig. 6.) When using the learning aid, the pupil should hold the beads that he is not using in one hand and manipulate the group that he is using with his other hand. If the child is permitted to keep a fact finder at his desk, he will have it available for handy reference with number stories and practice work.

The following outline is an illustration of how fact finders may be used in developing the addition fact that three and four are seven:

(1) Describe the necessary manipulations with the aid of the demonstration device.
(2) Ask the child to reproduce the set-up of the demonstration device on his own fact finder.
(3) Record the fact on the board in both of the following ways:
   (a) "Three beads and four beads are seven beads."
   (b) "Three and four are seven."
(4) Write the fact on the board vertically and horizontally in terms of the number symbols.
   (a)
$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$$
   (b) $3+4=7$.

Fact finders for use with second grade pupils should have 20 beads so that all combinations may be taught. The wooden frame for the demonstration device should be about 30 inches long in this case with the shield covering about half of the total length.

By using fact finders, pupils will be able to learn that addition is used when groups of beads are combined and that subtraction is used when groups are separated or compared. The devices can also be used to introduce pupils to the concept that combining equal groups of beads is a process of multiplication, while that of separating large groups into smaller groups is division.

Mrs. Edna Swan
Elementary Teacher
Parramore Tuberculosis Sanatorium
Lake County, Indiana

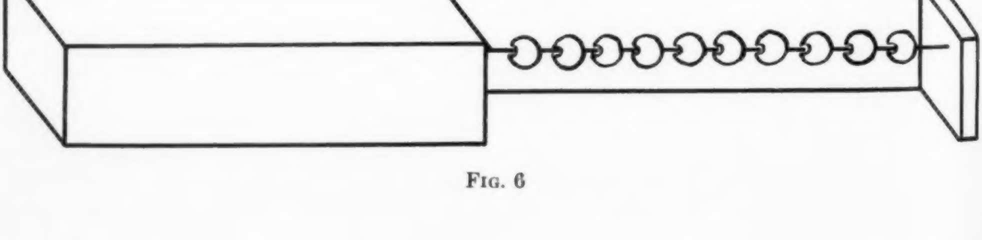


Fig. 6

APPLICATIONS

Edited by Sheldon S. Myers
University School, Ohio State University, Columbus, Ohio

With the last issue of the year, the summer vacation stretches ahead. We hope you will find time in the summer months to send us fresh ideas and examples for this department.

Ar. 8 Gr. 7–9 A Public Opinion Poll

When I asked my eighth grade mathematics class recently if they would like to take an opinion poll in their grade, they responded enthusiastically in the affirmative. The class spent about thirty minutes composing the questions which they wished to ask. They found that it was not easy to compose a question so that the replies would have clear meaning. Each word and phrase had to be carefully examined for ambiguity.

Finally they were satisfied with the following list of questions:

1. Should eighth graders date each other for school affiairs?
2. Should girls be permitted to wear blue jeans to school?
3. What time should eighth graders go to bed at night?
4. How many hours sleep a night should an eighth grader get?
5. What should be the weekly allowance of eighth graders?
6. Should eighth graders perform home tasks in order to earn their allowance?
7. Who do you think will be the next football coach at Ohio State University?
8. If you could have three wishes granted, what would you wish for?

After each question on the dittoed sheet the type of answer was indicated, such as, "yes," "no," "no opinion," a numerical reply, a name, or a sentence. When the raw data were presented to the class, a discussion arose as to how to treat the data in order to make it say more. It was decided to compute percentages on the yes-no type of answers and to make frequency tables on questions 3,4 and 5. Since each sheet was anonymous but marked boy or girl, the yes-no answers had three sets of data—boys, girls, and total. Quite a debate arose as to whether to compute the per cent of boys who replied "yes" to an item by dividing the number of boys into the number who said "yes" or by dividing the total in the class into the number of boys who replied "yes." Of course, there was no "right" way. After they had made the distinction in meaning between these two per cents, one-half decided to compute it one way and the other half the other way.

While they were working at their seats on these computations, many of them made some startling discoveries. For instance, when the same total group was being divided into two numbers, if one of the numbers was an even multiple of the other, the corresponding percentages had the same relationship. When all but one of a set of percentages had been computed, this one could be found by adding the others and subtracting from 100. Some students converted all of their division problems into multiplication problems as follows: there were 32 in the class; 1/32 was changed to a decimal which was then multiplied by each of the subgroups. All of these discoveries were eventually brought before the class and shared.

About a week later I learned that six allowances had been increased when the respective parents learned that their child's allowance was below the average of the class. During the class discussion on the data, the point was made that the size of an allowance did not mean much unless it were known whether such expenses as carfare and meals had to be paid out of it.

The latter point is an example of the type of interpretive thinking which the opinion poll fostered. Besides providing many valuable arithmetic insights, the opinion poll also stimulated critical thinking, provided strong motivation, and served a guidance function.

AL. 9 GR. 9–10 A FREQUENT PROBLEM INVOLVING POINT-HOUR RATIOS

Many 11th grade, 12th grade, and university advisers tell me that students often wish to know with what point-hour ratio they must finish a given block of work in order to have an over-all point-hour ratio above a stated minimum. For those of you who are unfamiliar with the method of computing a point-hour ratio, an illustration will be provided. A boy earns A's in two 3-credit hour courses; a B in a 5-credit hour course; and C's in three 2-credit hour courses.

Credit Hrs.		*No. of Courses*		*Total Hrs.*		*Grade Points*		*Total Points*
3	×	2	=	6	×	4 (A)	=	24
5	×	1	=	5	×	3 (B)	=	15
2	×	3	=	6	×	2 (C)	=	12
		0		0		1 (D)		
		0		0		0 (E)		
				17				51

$51 \div 17 = 3.0$ point-hour ratio.

Here is a sample problem involving point-hour ratios which algebra handles nicely.

For 178 hours a student has maintained a point-hour of 2.6. He wishes to know what average he must maintain for his last 18 hours in order to have the over-all average of 2.5 required for graduation.

Notice that the following equation involved in the solution has about the right degree of complexity for a verbal problem in ninth grade algebra.

$$18y+(178)(2.6)=2.5(178+18)$$

$$18y+462.8=490$$

$$18y=28.2$$

$$y=1.56$$ point-hour ratio required.

AR. 9 GR. 7–10 ARITHMETIC IN TRACK EVENTS

The May issue is a good time to run the following anonymous guidesheet used at the University School on the spring sport of track.

1. In a mile relay, the first boy ran his quarter in 53½ seconds, the second in 52½ seconds, the third in 54¾ seconds, and the last boy in 50½ seconds. How long did it take the team to run the race?
2. The American record for the mile run in 1929 was 4 minutes 12 seconds. The record for skating a mile was 2 minutes 45 seconds. How much shorter than the mile run time was this? The record time for swimming a mile was 21 minutes 46 seconds. How much longer than the mile run was this?
3. Sixteen times around he running track at Benton School make a mile. How many yards long is the running track?
4. Harry can run the 440 in 1 minute 10 seconds. How long would it take to run the mile at this rate?
5. In a contest Ray jumped 5′ 9″ while Joe jumped 6′ 2″. How many inches longer was Joe's jump than Ray's?
6. Jack won the 440 yard dash at the meet. What fraction of a mile is 440 yards?
7. In the Olympic games a man vaulted 3.9 meters. How many feet and inches was this?
8. How much difference is there between the 400 meter run and the quarter mile run? How many yards are covered in the 100 meter and the 800 meter events?
9. Which is the longer run and how much: 13 times around a quarter mile track or 7 times around a half mile track?
10. In a hop, skip, and jump event at a field meet, Donald hopped 3.1 feet, skipped 7.8 feet, and jumped 9.5 feet. What was Donald's total distance in this event?
11. In the 1936 Olympics, the 10,000-meter speed skating contest was won by Ballagrud of Norway in 17 minutes 24.3 seconds. The American record for six miles at that time was 18 minutes 38 seconds held by Johnson of Montreal. How much faster per mile was the Olympic record than the American record?
12. Bob ran 220 yards in 27.5 seconds. What was his average speed in yards per second and in miles per hour?

S. G. 2 GR. 11–14 SIMILARITY OF FORM OF EULER'S FORMULA AND GIBBS' PHASE RULE

There are many times in mathematics and science when the mind pauses in silent wonder. Duality, the principle of

isomorphism between logical calculi and their interpretations in mathematics and science, the role of the calculus in mechanics, the Euler Line and the Nine-Point or Feuerbach Circle, the fixed-point theorem in continuous geometric transformations without translation—these are some of the things which generate simultaneously a feeling of awe, admiration and humility. One of these things which is not generally known or publicized is the similarity between Euler's Formula and Gibbs' Phase Rule. Here are the two formulas:

Euler: $V=E-S+2$
Where V = no. of vertices
E = no. of edges
S = no. of surfaces of any polyhedron.

Gibbs: $F=C-P+2$
Where F = no. of degrees of freedom
C = no. of components
P = no. of phases of any system of equilibrium.

The Phase Rule is one of the great generalizations of modern science and represents the foundation and corner stone of thermodynamics and such industrial fields as metallurgy, oil industry, and synthetic organic chemistry. The Phase Rule deals with such systems of equilibrium as water in a tight container with vapor above it in equilibrium with the liquid. Degrees of freedom refer to such variables as temperature, volume, and pressure. Components refer to the number of distinct chemical substances present, such as water. Sugar dissolved in the water would represent a second component. Phases refer to the number of states of matter present in equilibrium, such as gas, liquid and solid. These three are the maximum number of states possible. Sometimes a substance can have several solid phases such as sulfur, which has two crystalline and one amorphous solid phases.

The Phase Rule can be used to predict the number of degrees possible in any given system of equilibrium. For example, if water and water vapor were present in equilibrium, $C=1$, $P=2$. Therefore:

$$F=1-2+2=1\cdot$$

This means that temperature, or pressure, or volume may vary and equilibrium retained, but not more than one of these.

If ice, water, and water vapor were present in equilibrium, $C=1$, $P=3$. Therefore:

$$F=1-3+2=0\cdot$$

This means that ice, water, and vapor can be present in equilibrium only at a fixed temperature and pressure and volume, because the Phase Rule predicts that there are no degrees of freedom in this system of equilibrium. The specific temperature required for the equilibrium of the three phases of water is known as the "triple point" for water.

When the two axes of a graph are allowed to represent the two degrees of freedom of a system of equilibrium and the phases in equilibrium mapped out on the coordinate system, a "phase diagram" is created. Phase diagrams for thousands of substances and mixtures have been worked out experimentally. The theory which has been developed about such diagrams is known as the "thermodynamics of plane surfaces." When three degrees of freedom are plotted on three axes, three dimensional phase diagrams are created. While on the plane diagrams the boundaries between phases are represented by curved lines, on three dimensional phase diagrams, the boundaries of phases are represented by surfaces. These surfaces intersect along edges, which come together at vertices. This would indicate that some kind of isomorphism has been established between the V, E, and S in polyhedra and F, C, and P in systems of equilibrium. The relation stated by these two formulas seems to be a topological one.

We would appreciate receiving from the readers any further information about these two formulas and their relationship.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* PAUL M. WEISEL
The University of Wisconsin, Madison, Wisconsin

SECTION A of a study on trends in teaching mathematics was published in the November issue of THE MATHEMATICS TEACHER. A summary of replies to the questions in Section A, which was concerned with mathematics enrollments, was reported in the April number. Because of the small number of replies received the questions of Section A will be repeated in THE MATHEMATICS TEACHER for October.

Section B, General Mathematics, appeared in the February number of THE MATHEMATICS TEACHER.

Questions in Section C are on content and practice in the third year of sequential mathematics. It will be interesting to determine if there is a definite trend to complete the third year in any of the various ways listed: more algebra, solid geometry, or trigonometry. Requests for information on teaching aids has been limited to films and filmstrips in keeping with the policy of this section to be brief and to include only questions on which answers can be easily reported.

It is hoped that this question-answer department will yield information on trends in teaching mathematics which will be useful in the study and research carried on by teachers everywhere. The success of this project depends on the co-operation of readers of THE MATHEMATICS TEACHER. Teachers are invited to submit questions for publication in future issues.

In replying please give your name, the name and locality of your school, and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades 10–12; junior high school, grades 7–12; etc. The questions may be answered by numbers without use of a particular form or repetition of the questions. The names of teachers or schools will not be revealed in any of the reports.

Please send the answers to these questions to John R. Mayor, North Hall, Madison 6, Wisconsin by June 10, 1951.

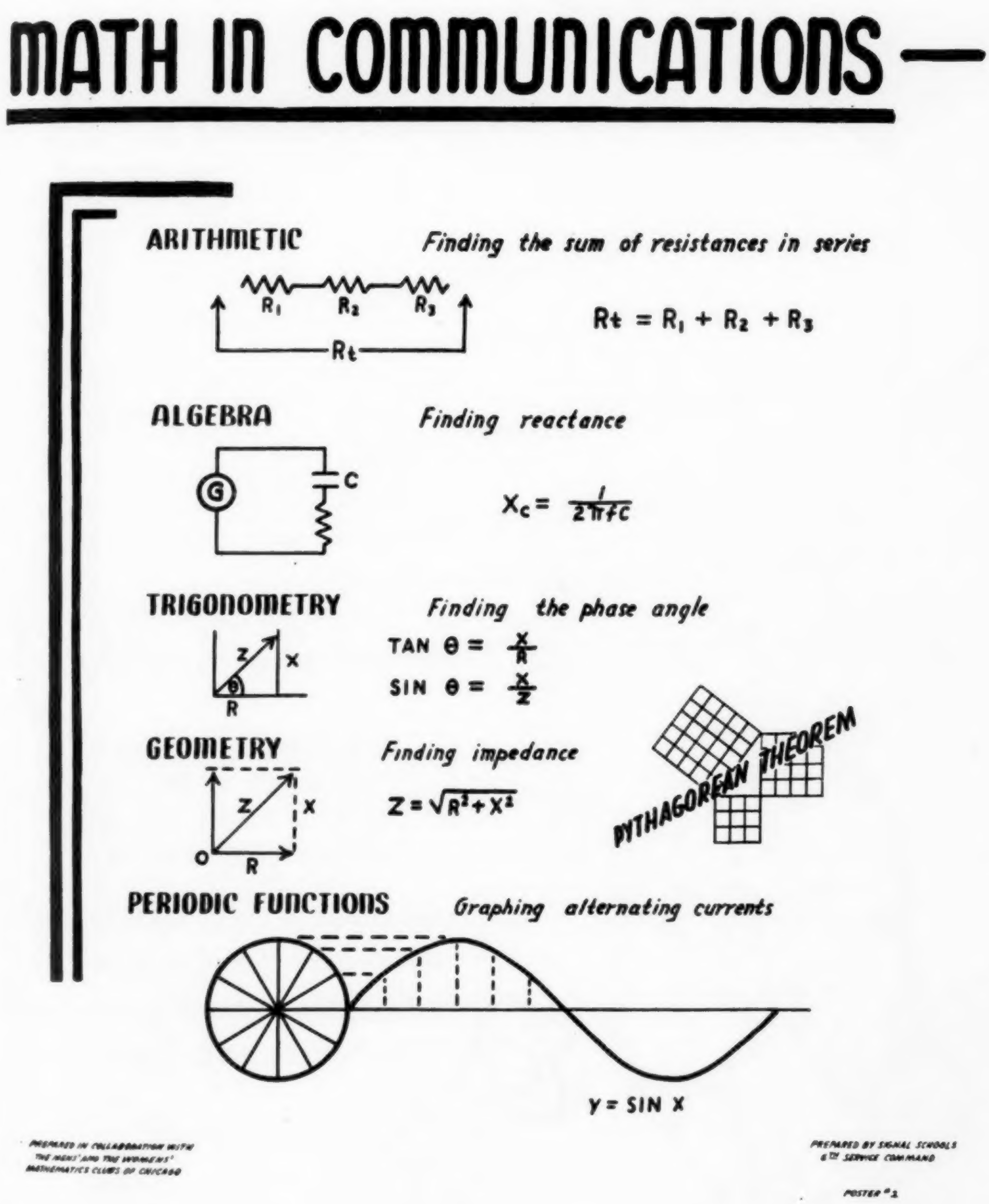
C. THIRD YEAR MATHEMATICS

1. What courses do you offer for the third year of sequential mathematics?
   a. A third semester of algebra and a semester of solid geometry _____
   b. A third semester of algebra and a semester of trigonometry _____
   c. A second year of algebra _____
   d. A year of plane geometry _____
   e. Algebra and another subject in combination different from that in a) or b); if e) is checked, please indicate also _____
      Number of weeks of algebra _____
      Number of weeks of solid geometry _____
      Number of weeks of trigonometry _____
   f. Other (please describe briefly).
2. Do you have ability grouping in your third year course? _____
3. Do you have a mathematics club:
   For juniors only? _____
   For seniors only? _____
   For both juniors and seniors? _____
4. Do you use films and film strips in the third year of sequential mathematics? _____
   If so, please list the two you have found most helpful.
5. Do your third year mathematics students use the library as part of the suggested study in this mathematics course? _____

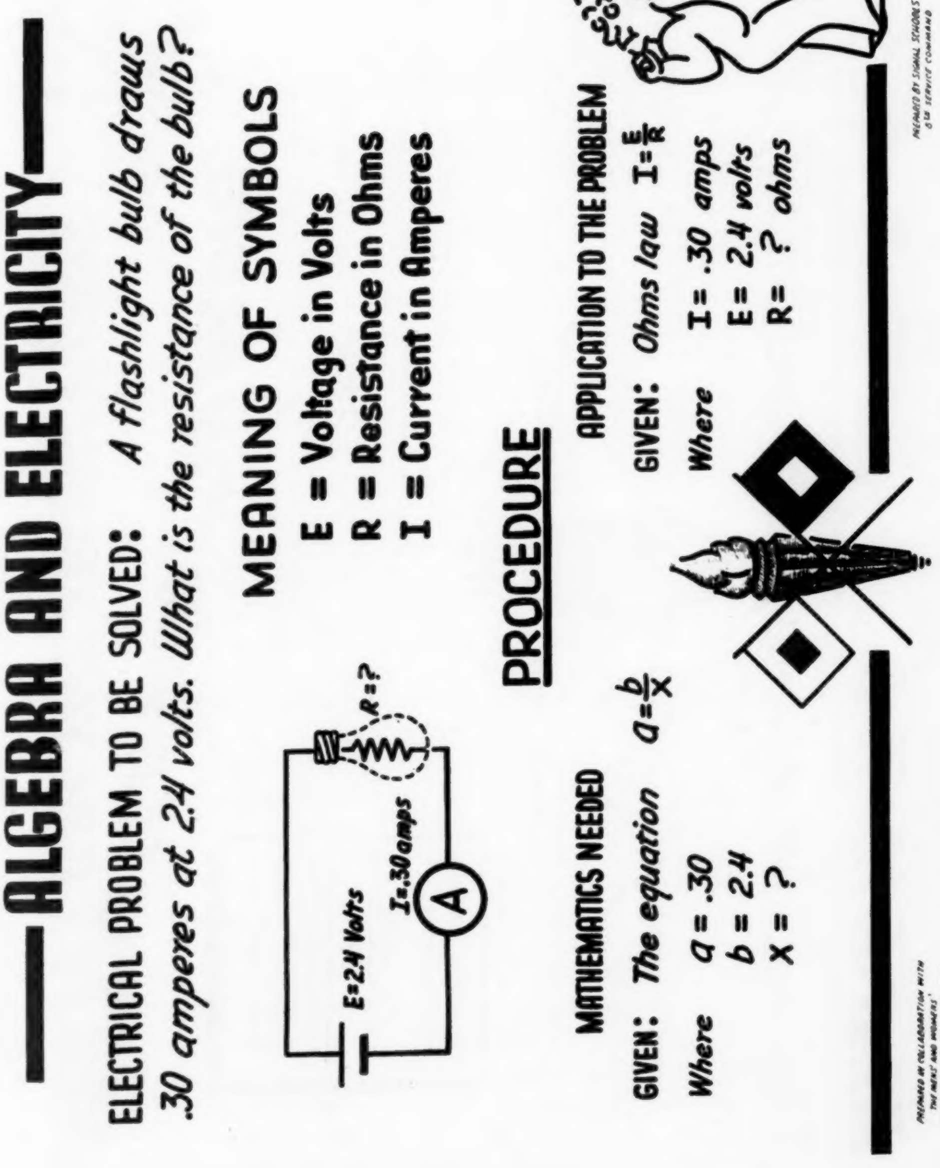
Mathematics Kits

Beginning July 1, 1951, all orders for mathematics kits should be sent to the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C. For the past two years, the kits have been distributed by Mr. M. H. Ahrendt, Anderson College, Anderson, Indiana, and he will continue to receive orders until July 1. After that date, the distribution will be handled by the Washington office of the Council.

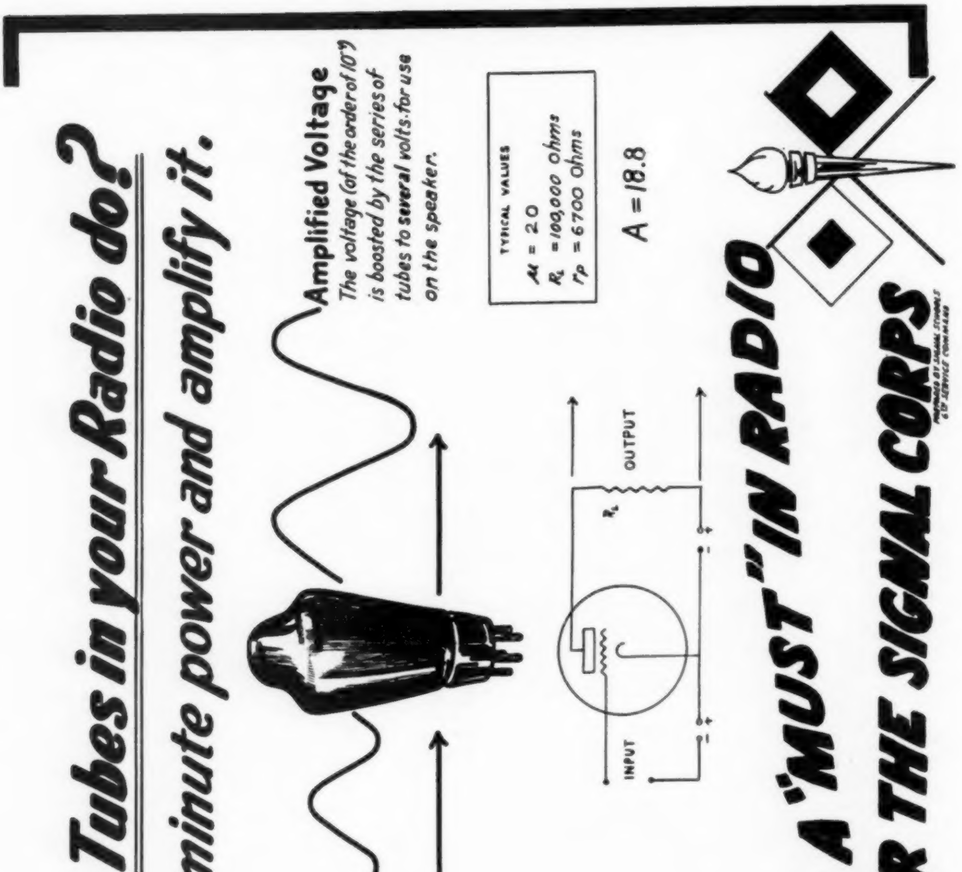
Computation Unit No. 123 is proving very popular. Many teachers have ordered this kit in quantities for use in entire classes. Straight Line Unit No. 113 is still available. All kits sell for 50¢ each or 3 for $1.00. Quantities may be assorted.


MATH IN COMMUNICATIONS —
ARITHMETIC
Finding the sum of resistances in series
R1 R2 R3
Rt
Rt = R1 + R2 + R3
ALGEBRA
Finding reactance
G
C
Xc = 1/2πfc
TRIGONOMETRY
Finding the phase angle
Z X R θ
TAN θ = X/R
SIN θ = X/Z
GEOMETRY
Finding impedance
Z X O R
Z = √(R² + X²)
PYTHAGOREAN THEOREM
PERIODIC FUNCTIONS
Graphing alternating currents
y = SIN X
PREPARED IN COLLABORATION WITH THE MEN'S AND THE WOMEN'S MATHEMATICS CLUBS OF CHICAGO
PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND
POSTER #1


PREFIXES
USED IN ELECTRICITY
MEGA [M] means 1,000,000 times
KILO [k] means 1,000 times
CYCLE-OHM-VOLT-AMPERE-FARAD-HENRY
MILLI [m] means $\frac{1}{1000}$ times
MICRO [μ] means $\frac{1}{1,000,000}$ times
MICRO MICRO means $\left[\frac{1}{1,000,000}\times\frac{1}{1,000,000}\right]$ times
FARAD
WHEN FUNDAMENTAL UNITS, SUCH AS THE AMPERE OR OHM, ARE TOO LARGE OR TOO SMALL FOR CONVENIENCE A PREFIX IS USED TO GIVE A UNIT OF PROPER SIZE.
$\frac{1}{1000}$ ampere = 1 ma [1 milliampere]
1000 watts = 1 Kw [1 Kilowatt]
$\frac{25}{1,000,000,000,000}$ farads = 25 μμf [25 micro micro farads]
3,500,000 cycles = 3500 Kc = 3.5 Mc
I'll mow him down to size!!
μμ
PREPARED IN COLLABORATION WITH "THE MEN'S" AND THE WOMEN'S" MATHEMATICS CLUBS OF CHICAGO
PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND
POSTER # 2



ALGEBRA AND ELECTRICITY
ELECTRICAL PROBLEM TO BE SOLVED: A flashlight bulb draws .30 amperes at 2.4 volts. What is the resistance of the bulb?
E=2.4 Volts
I=.30 amps
A
R=?
MEANING OF SYMBOLS
E = Voltage in Volts
R = Resistance in Ohms
I = Current in Amperes
PROCEDURE
MATHEMATICS NEEDED
GIVEN: The equation a=b/x
Where a = .30
b = 2.4
x = ?
APPLICATION TO THE PROBLEM
GIVEN: Ohms law I=E/R
Where I= .30 amps
E= 2.4 volts
R= ? ohms
PREPARED IN COLLABORATION WITH THE MEN'S AND WOMEN'S MATHEMATICS CLUBS OF CHICAGO
PREPARED BY SIGNAL SCHOOLS 6th SERVICE COMMAND



What do the Tubes in your Radio do?
They receive minute power and amplify it.
A small Incoming Signal Voltage
About 10^{-4} Volts
Amplified Voltage
The voltage (of the order of 10) is boosted by the series of tubes to several volts for use on the speaker.
Formula for Computing "GAIN" of tube:
$A = \frac{\mu R_L}{T_p + R_L}$
R_L = "Load Resistance" (in Ohms)
T_p = "Plate Resistance" (in Ohms)
μ = "Amplification Factor" of tube
A = "Gain" of tube
INPUT
OUTPUT
TYPICAL VALUES
μ = 20
R_L = 100,000 Ohms
r_p = 6700 Ohms
A = 18.8
MATH IS A "MUST" IN RADIO
FOR THE SIGNAL CORPS
PREPARED IN COLLABORATION WITH THE MEN'S AND THE WOMEN'S MATHEMATICS CLUBS OF CHICAGO
PREPARED BY SIGNAL SCHOOLS 6TH SERVICE COMMAND

THE PRESIDENT'S PAGE

What are the recent developments in the mobilization of education for national security? What is the impact of mobilization on instruction in the schools? What are the implications for mathematics?

The National Conference for Mobilization of Education (MOE) was established at a meeting held in Washington on September 9 and 10 by representatives of major national educational organizations who met with representatives of the Armed Services and the Federal Government to review the experience of World War II as it relates to schools and colleges, to study the present aspects of the emergency situation with particular emphasis on the relationship to education, and to develop long-range plans for continuous coordinated educational effort in support of the national security. The purpose of MOE is to further the cooperative efforts of voluntary educational organizations in the mobilization of the nation, each participating organization to retain full freedom of action with respect to its own policies and programs. It is the aim of MOE to strengthen rather than restrict the operations of the participating organizations. Your National Council is one of the more than eighty participating organizations in MOE. Each organization has two representatives. The National Council is being represented at present by Professor Walter H. Carnahan of Purdue University and your president. The MOE is to serve as a cooperative channel of communication between organized education in the United States and the Federal Government on matters pertaining to mobilization of education.

On January 26 and 27 a work conference dealing with the impact of mobilization on instruction in the schools was held in Washington. Your president represented the National Council at this conference in which sixty school and military people participated. The purpose of the conference was to clarify objectives and to help strengthen and improve instructional programs in this mobilization period.

The following recommendation came from the conference of January 26 and 27: "This conference requests the National Conference for Mobilization of Education to recommend strongly that the Office of Defense Mobilization and Office of Defense Manpower in the allocation and assignment of personnel under the defense program give every consideration to the necessity for retaining as far as possible in elementary and secondary schools the teaching personnel necessary for the adequate education of all children and youth." And these relating to mathematics and science: "This conference believes that the programs of mathematics and science in the elementary and secondary schools that contribute best in times of peace likewise contribute best in times of stress and conflict. Therefore we urge, (1) that all children and youth be offered the opportunity and be encouraged to develop competency and power in science and mathematics to the extent of the individual's ability, (2) that re-emphasis be placed on basic skills, meanings, and understandings, and (3) that specific preinduction courses in mathematics and science *not* be introduced until their value has been determined by the cooperating organizations of MOE and the Federal Agencies working through the U. S. Office of Education."

There is no call for schools to set aside any part of a sound curricula or school program; but it is urgent that present programs and courses be re-examined to check on permanent values. At present, the military people are not asking for specialized or pre-induction courses in the high school. They do advise, however, that children and youth get a sound and substantial training in basic understandings and fundamental skills.

H. W. Charlesworth,
President

Organization Patterns of Affiliated Groups

JOHN R. MAYOR,
Chairman, Committee on Affiliated Groups

LEADERS of organizations of mathematics teachers considering affiliation with the National Council ask many questions about the nature of the organization of Groups already affiliated. The Committee has now on file constitutions of the various Affiliated Groups and these are made available to any Groups, affiliated or unaffiliated, which are writing their first constitutions or considering revisions. The constitutions are the best source of information on the nature of the various organizations and are the source of most of the information in this report.

REGIONAL ORGANIZATION

The regional bases of organization of the 43 Groups which had completed affiliation by January 1, 1951, are:*

City—11
[Chicago (Men, Women, Elementary), Cleveland, Dallas, Detroit, New York, Oklahoma City, Richmond, Tulsa, Wichita]

County—4
[Dade and Hillsborough, Florida; Nassau and Suffolk, New York]

Sectional within a state—4
[Eastern Colorado, Southern Oregon, Western Pennsylvania, Eastern Tennessee]

State and Province—22
[Alabama, Arizona, Arkansas, California, Colorado, Florida, Georgia, Illinois, Indiana, Iowa, Kansas, Kentucky, Maryland, Minnesota, Nebraska, New Jersey, Oklahoma, Ontario, Texas, Virginia, West Virginia, Wisconsin]

Sectional including more than one state—2
[Louisiana-Mississippi, New England]

Of the eleven city organizations eight are in states which also have a state-wide affiliated organization. The remaining three are in Cleveland, Detroit, and New York. These cities are in states where a state-wide organization, not yet affiliated, has either recently been formed or is in process of formation.

Two of the county groups are in Florida which has a state-wide Affiliated Group while the other two are in New York, where much progress is being made on a state-wide organization. It should also be pointed out that a third Florida county has previously been affiliated and probably will have renewed its affiliation by the time this report is published.

The four sectional organizations within a state are in Eastern Colorado, Southern Oregon, Western Pennsylvania, and Eastern Tennessee. Colorado also has a state-wide affiliated organization and Pennsylvania probably will have by the time of the Pittsburgh meeting. A new organization in Western Tennessee has taken the first steps to complete affiliation.

States with more than one Affiliated Group are:

	No. of Groups
Colorado	2
Florida	3
Illinois	4
Kansas	2
New York	3
Oklahoma	3
Texas	2
Virginia	2

Of the 22 state-wide Affiliated Groups, including Ontario, 14 are in states which do not also have another Affiliated Group within the state. In the March issue of THE MATHEMATICS TEACHER, a request was made for assistance in interesting the remaining states in affiliation. These states, 13 in number, are Delaware, Idaho, Missouri, Montana, Nevada, New Mexico, North Carolina, North Dakota, South Carolina, South Dakota, Utah, Washington, and Wyoming. Interest in affiliation on the part of mathematics teachers in two of these states has already been reported in this section in recent

* NOTE: The official list for the Second Delegate Assembly included 50 Groups. Seven Groups completed affiliation between January 1 and March 29, 1951. These Groups are:
City—Philadelphia, Washington, D. C. (Benjamin Banneker Club)
County—Pinellas, Florida
State—Pennsylvania, South Carolina, South Dakota, Utah.

issues and this spring correspondence with two others has been encouraging.

Two of the most successful of all Affiliated Groups have been the Association of Teachers of Mathematics in New England and the Louisiana-Mississippi Branch of the National Council. In these areas teachers have found advantages in a larger organization which crosses state lines.

A question often discussed is whether it might be better for the National Council to recognize only state-wide groups or groups made up of several states as Affiliated Groups and then to encourage these organizations to recognize in a similar manner city, county, and regional groups. It seems doubtful that a required pattern of this type will ever be desirable. There are such great variations in distribution of population among the states that what works most efficiently in one area cannot be expected necessarily to be best for another. Those responsible for the organization of any new group should give careful consideration to the various possibilities, and should also, if possible, study constitutions of and communicate directly with Affiliated Groups with organizations of all of the types under consideration.

This study has revealed even more clearly than was previously realized that strong city and county organizations have had a great deal to do with the establishment of state organizations and are continuing to contribute much to their support. These city and county organizations are likewise continuing to contribute directly to and to gain directly from the National Council through their affiliation.

Relationships With Other Associations

Many of the Affiliated Groups are associated with their state education association in a relationship similar to that they bear to the National Council. Among these are the state groups in:

Alabama	New Jersey
Arkansas	Pennsylvania
Colorado	Tennessee
Florida	Texas
Georgia	Virginia
Iowa	West Virginia
Maryland	Wisconsin

In most instances these Affiliated Groups are either the mathematics sections of the state education association, or organizations which have grown from these sections. An opportunity for important and fruitful endeavor is certainly provided by such a relationship. This was well brought out in the panel discussion on Affiliated Groups at the Florida meeting of the National Council.

In a number of instances in which Affiliated Groups are also associated with their state education association this relationship has provided the basis for the state organization to maintain special relationships with the mathematics sections of the divisional education associations within the state.

Two of the regular meetings of the Association of Mathematics Teachers of New Jersey are held in co-operation with larger state educational associations. In November the Association meets at Atlantic City as an Affiliated Group of the New Jersey Education Association and in May, at Rutgers University, New Brunswick, in connection with the New Jersey Secondary School Teachers Association. New Jersey is one of the first of the state Groups to include chairmen of sectional groups on its Executive Board.

In the very comprehensive annual report of the New Jersey Association, prepared by Mary C. Rogers, one of the factors listed as contributing to the excellent growth of the New Jersey Group was the co-operation of the State Department of Education with the Association. This co-operation has been shown both in special studies and problems of educational research and in matters pertaining to membership growth, such as supplying periodically lists of teachers of mathematics within the state.

The Indiana Council of Teachers of Mathematics recognizes five regional groups and local affiliated mathematics clubs as important components of the state-wide organization. A diagram showing the relationships within the Indiana organization was included in the second issue of the Newsletter on Affiliated Groups.

The California Council, in adopting a new constitution two years ago, was particularly interested in developing a "unified state organization" and in determining that "local initiative and activity shall be stimulated." The Council realized the importance of local interests and the need for the discovery and development of leadership ability. In the new organization, functioning in close harmony with the president, vice-president-secretary, and treasurer of the State Council are six additional State Executive Board members, who are the officers of the Northern and Southern Sections of the state. The directors of the Northern and Southern Sections are aided by a secretary and treasurer for each Section and the chairmen of various "sub-groups" that are organized in "naturally related areas." These chairmen, with three sectional officers, form the board of directors for each section. It follows quite naturally that one of the more important functions of this board is to exchange ideas on programs and activities for all sections and sub-groups.

The constitution of the Wisconsin Mathematics Council provides for an executive committee to be composed of the officers of the Council, the presidents of various state divisional education associations, presidents of any city or area mathematics clubs with at least 35 members, the past president of the Wisconsin Mathematics Council, and the state representative of The National Council of Teachers of Mathematics.

Many other groups have planned programs intended to promote more and better cooperation with local organizations. The Dade County (Florida) Council of Teachers of Mathematics sponsors a curriculum committee which coordinates its work with the various curriculum agencies of Dade County and with the Florida State Department of Education in discussing and formulating curriculum policy.

The Association of Teachers of Mathematics in New England has provided in its constitution for the "organization of sections on the basis of geographical location of members of the association," these sections being authorized to hold meetings at their own discretion.

The Kentucky Council of Mathematics Teachers keeps in contact with other interested persons by sending out notices of meetings to districts where mathematics teachers are not organized.

The Iowa Association of Mathematics Teachers maintains an advisory board which considers policies proposed for adoption by other groups interested in mathematics and assists and advises chairmen of the district mathematics sections in planning programs for district meetings. The annual meeting of the Iowa Association is held in connection with the Iowa State Education Association.

While the general pattern of organization is for Affiliated Groups to include among their members teachers of mathematics from all levels of instruction, there are two Groups which are concerned primarily with instruction in mathematics at the elementary school level. These are the Chicago Elementary Teachers Mathematics Club and the Dallas Elementary Mathematics Association. Because we often fail in mathematics organizations to offer as much to teachers at the elementary level as we are able to give at the secondary school level, these Groups should have a special welcome. The state-wide Affiliated Groups in Illinois and Texas are able to operate more effectively at the elementary school level because of strong, active Groups for elementary teachers within their area of responsibility.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Membership Record

MARY C. ROGERS, *Roosevelt Junior High School, Westfield, N. J.*

100% Schools—as of March 20, 1951

1. Tampa, Florida .... Brandon High School
2. South Bend, Indiana .... James Madison Junior High School
3. Elizabethtown, Pennsylvania .... Elizabethtown College
4. Gray Court, South Carolina .... Hickory Tavern High School
5. Richmond, Virginia .... John Marshall High School
6. Washington, D. C. .... McKinley High School

"All but One" Schools—as of March 20, 1951

1. Penns Grove, New Jersey .... Regional High School
2. Washington, D. C. .... Phelps Vocational High School
3. Washington, D. C. .... Taft Junior High School

We have prepared for you a few interesting membership data in addition to the listing of the Membership Honor Schools. Much more information of this sort is available and may be released to you from time to time. Our present *membership total* is *8479*.

TEN STATES LEADING IN MEMBERSHIP TOTALS

1. Illinois	710	6. California	348
2. New York	671	7. Indiana	324
3. Pennsylvania	550	8. New Jersey	321
4. Texas	387	9. Michigan	275
5. Ohio	364	10. Massachusetts	272

STATES SHOWING CONTINUOUS GROWTH

1947–1951

Delaware	Indiana	Tennessee
Florida	Maryland	Texas
Georgia	Pennsylvania	Virginia

STATES WHOSE MEMBERSHIP HAS MORE THAN DOUBLED

1947–1951

Arizona	Idaho	Nevada
Delaware	Indiana	New Mexico
Florida	Louisiana	Utah
Georgia	Mississippi	Virginia

STATES WITH 50%-100% INCREASE IN MEMBERSHIP

Maryland	Oklahoma	Texas
New Hampshire	Tennessee	West Virginia

Our congratulations to each of you for this excellent record. Please accept our sincere thanks for your outstanding support. We are deeply grateful for the enthusiastic cooperation which you continuously give us in the many professional services which we are undertaking together.

KNOW YOUR NATIONAL COUNCIL REPRESENTATIVE

By KENNETH E. BROWN
University of Tennessee, Knoxville, Tennessee

MISS MAUDE C. HOLDEN

NEBRASKA REPRESENTATIVE

MISS MAUDE C. HOLDEN, a high school teacher at Ord, Nebraska, has recently assumed the responsibilities of National Council Representative for Nebraska. Miss Holden is a Past President of the Nebraska Section of the National Council Council of Teachers of Mathematics and she gave favorable reports of the activities of the Nebraska Section at the National Council meetings at Denver, Colorado, and Madison, Wisconsin. She is looking for energetic key members to help plan an intensive campaign in Nebraska for new National Council members.

SOUTHERN CALIFORNIA REPRESENTATIVE

DALE CARPENTER, the National Council Representative in Southern California, is also the Mathematics Supervisor of the junior and senior high schools in Los Angeles. His experience in teaching for twelve years in the junior high school in addition to a similar period as a supervisor of high school mathematics has given him an invaluable background for his present position. Mr. Carpenter has had a special interest in improving the mathematics for the non-college preparatory student. This interest has resulted in the basic mathematics courses in the Los Angeles high schools for those students not preparing for college. His enthusiasm for better mathematics teaching has contributed greatly to the steady increase in the National Council membership in California.

DALE CARPENTER

Texas Representative

Miss Mary Ruth Cook, as a member of the faculty of North Texas State College, conducts the courses in the teaching of mathematics and supervises the student teachers in the Laboratory School of the College. Her interest in professional improvement is indicated by the fact that she is a member of the National Education Association, National Association of Student Teaching, College Classroom Teachers Association, Kappa Delta Pi, Delta Kappa Gamma, Delta Psi Kappa, a life member of the Texas State Teachers Association and the Representative of the National Council since 1945.

Since Miss Cook has been the National Council Representative, the membership has increased from 200 to approximately 400. For this increase in membership she gives the credit to her twelve Regional Representatives. However it has required considerable effort to secure the help of these energetic key teachers and personally send out hundreds of invitations to join the National Council. According to Miss Cook, a sample copy of The Mathematics Teacher is one of the best means of publicity for the National Council of Teachers of Mathematics. In a recent letter she states, "I am particularly pleased with the issues of The Mathematics Teacher to date this year."

Miss Mary Ruth Cook

Miss Eva Burkhalter

Oregon Representative

Miss Eva Burkhalter is not only a native of Oregon but, with the exception of advanced graduate work at the University of Washington, her academic education was received in Oregon. Miss Burkhalter has devoted her professional career to the improvement of the teaching of mathematics on the junior and senior high school level. During her ten years at Klamath Union High School, she has done much to realize this goal.

Miss Burkhalter's interest in the National Council began when her teacher gave her a copy of The Mathematics Teacher. Since that day, she has read all the copies of The Mathematics Teacher from Volume 1, Number 1, to the present issue. Also she states that the Yearbooks have been most helpful. Miss Burkhalter has been the Oregon Representative for two years and she is seeking key teachers who will help her in the National Council.

MATHEMATICS FOR NATIONAL SECURITY

THE events of the past few months have reminded us of similar ones which took place in the late thirties and the early forties. During that period it was necessary to enlist the cooperation of mathematics teachers in every part of the country. Today, the need for such assistance is even greater.

Special emphasis has been placed by those concerned with our country's semi-mobilization needs on securing personnel with an amount of training far exceeding that ever required before in our history. Since much of this training parallels that which was found necessary in the period referred to above, we wish to call attention to a few of the important recommendations made by committees and members of the National Council in earlier issues of THE MATHEMATICS TEACHER. These statements and many others are among those which call for our most careful consideration today.

From "Pre-Induction Courses in Mathematics" on page 115 of the March 1943 issue:

> The emergency need for boys and girls trained in mathematics has focused attention on the highly technical features of our mechanized civilization. The armed services and the supporting war industries need boys and girls trained in the proficient use of mathematics ranging from a real mastery of arithmetic fundamentals and such practical uses as are found in courses in general mathematics to the uses of higher mathematics in meteorology, ballistics, and other branches of science. Girls trained in mathematics are needed to replace men in industrial and other civilian positions which require the same range of uses of mathematics.

From "Progress Report of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America" on page 301 of the November 1941 issue:

> In the junior and senior high schools,each boy and girl *of sufficient mathematical aptitude* should be urged by his advisers to observe that the study of mathematics through the stage of trigonometry and some solid geometry may serve as a distinctly patriotic action.

From "The First Report of the Commission on Post-War Plans" on page 231 of the May 1944 issue:

> The simplicity of mathematical skills needed by the masses in the armed forces is even greater than was generally assumed, but is, as has been pointed out elsewhere, very difficult for many persons to achieve. Provision for growth in the mastery of arithmetic should be continuous throughout the elementary and secondary schools. Achievement of ability in mathematical reasoning and in sensible use of mathematical concepts requires much time and continuous practice. Courses of study of the elementary school should be reconsidered and adequate time be given to the teaching of arithmetic. Moreover, we have not given enough attention to arithmetic in teaching the senior high school courses. In fact, we have too often allowed the fundamental skills to deteriorate. Hereafter, we should keep an eye on arithmetic as we teach the advanced courses. Finally, the administrator can't eat his cake and have it. If we are to be held for results, we need to have the students for whatever time it takes to do the job.

From "Influence of the War on the Teaching of Secondary Mathematics" by E. R. Breslich on pages 293 and 294 of the November 1944 issue:

> Proficiency in arithmetic should be a requirement for graduation from the high school.
>
> The teaching of all high school mathematics should stress understanding of concepts and principles.

From "Mathematical Education in War-Time" by Virgil S. Mallory and Howard Fehr on page 298 of the November 1942 issue:

> The army, navy, industry, and scientific organizations indicate that the mathematics teachers can be of immediate aid in the war emergency of securing the following:
>
> 1. Every boy in high school should study mathematics according to his abilities, the more capable taking four years of mathematics including trigonometry and solid geometry.
> 2. Advisers must realize the acute need for mathematics and direct all capable boys into such courses.
> 3. It is not necessary to create new courses or to entirely reorganize the present courses in the wartime effort. If our objectives are valid, and our methods of teaching will obtain these objectives, all that is needed is an added emphasis on the practical. . . .
> 5. Good teaching of straight mathematics

by fully qualified teachers is what is needed. Such teaching is a far superior procedure than to attempt courses in aeronautics or navigation in the high school given either by teachers not fully qualified to teach or taken by pupils who have not the necessary foundations for their successful study.

6. Modify your subject matter and your rate of teaching to attain complete understanding and you will best fit boys and girls for both peace-time and war-time uses of mathematics . . . .

From "The Second Report of the Commission on Post-War Plans" in the May 1945 issue:

The school should guarantee functional competence in mathematics to all who can possibly achieve it.

We must discard once for all the conception of arithmetic as a mere tool subject.

We must conceive of arithmetic as having both a mathematical aim and a social aim.

We must give more emphasis and much more careful attention to the development of meanings.

We must abandon the idea that arithmetic can be taught incidentally or informally.

We must realize that readiness for learning arithmetical ideas and skills is primarily the product of relevant experience, not the effect of merely becoming older.

We must evaluate learning in arithmetic more comprehensively than is common practice.

The mathematical program of grades 7 and 8 should be essentially the same for all normal pupils. (a) It should provide an adequate, organic continuation of the work of grades 1–6. (b) It should provide a substantial beginning in achieving functional competence. (c) It should provide a dependable foundation for subsequent courses in mathematics. (d) It should be so organized as to enable the pupils to achieve mathematical maturity and power.

The large high school should provide in grade 9 a double track in mathematics, algebra for some and general mathematics for the rest.

The main objective of the sequential courses should be to develop mathematical power.

Nominations for Officers and Members of the Board of Directors

The By-Laws of the National Council of Teachers of Mathematics require that "the Nominations and Elections Committee shall cause an announcement to be published in the official journal at least five months before the annual meeting inviting members of the Council to suggest nominees for elective offices."

The 1952 ballot will present candidates for the following offices:

A President (to serve for two years)
A Vice-President representing elementary school mathematics (to serve for two years)
A Vice-President representing senior high school mathematics (to serve for two years)
A Vice-President representing junior high school mathematics (to serve for one year only. This is a new office created by the revised By-Laws, for which election will thereafter be held at two-year intervals.)
Three members of the Board of Directors (to serve for three years)

The candidates for Directors may represent any level of instruction. The present geographic plan requires that only one of the nine Directors may come from one state. This limitation does not apply to officers. Hence, the following states will not be represented on the ballot for the three Directors: Louisiana, Minnesota, New Jersey, Florida, Kansas, and Iowa.

The Nominating Committee invites members of the Council to suggest persons to be nominated and to submit brief statements of their qualifications and their participation in local or national affairs pertaining to mathematics. Suggestions should reach the committee as soon as possible, and no later than September 1, 1951. Suggestions may be sent to any member of the Nominating Committee: E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana; Ona Kraft, Collingwood High School, Cleveland, Ohio; Veryl Schult, Wilson Teachers College, Washington, D. C.; Lenore John (Chairman), Laboratory School, University of Chicago, Chicago, Illinois.

The Twelfth Christmas Meeting of the National Council of Teachers of Mathematics will be held December 27, 28, 29, 1951, at the new Student Union Building on the campus of The Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma. The Program Chairman is Miss Lenore John, Laboratory Schools, University of Chicago, Chicago, Illinois and the Local Chairman is James H. Zant, Department of Mathematics, Oklahoma A. and M. College, Stillwater, Oklahoma. Plans for the tentative program include a general session on Friday and on Saturday morning at 9:00 A.M. as well as sectional meetings at 10:30 and in the afternoon on both days.

TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

The Correlation of Mathematics and Science

FIFTY YEARS ago the Englishman John Perry advocated greater emphasis upon the practical uses of mathematics arising in mechanical drawing, physics, chemistry and engineering. A quarter of a century later, the National Committee report (1923) in this country recommended (p. 28) that there should be a conscious effort, through the selection of problems, to correlate the work in mathematics with other courses in the curriculum, especially in science. In 1942 the National Council of Teachers of Mathematics published as its 17th Yearbook, *A Source Book of Mathematical Applications*. Despite the wealth of excellent material assembled in that yearbook, to what extent are mathematics and science being correlated *systematically* and *effectively* in typical mathematics courses throughout the country today?

In view of the crucial role played by science and technology in contemporary civilization, and because of the intimate relationship between science and mathematics, both pure and applied, it would seem that a more adequate correlation can and should be effected between the two disciplines. It has been possible to integrate them so thoroughly that the casual visitor could not decide whether the lesson he was observing was one in mathematics or in science. Such complete correlation probably need not be our goal, but present practice certainly leaves much to be desired. To facilitate improvement in this connection, the following source material has been collected in the hope that it may prove helpful to teachers, curriculum committees, authors of textbooks, and mathematics clubs.

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MATHEMATICAL RECREATIONS

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THE GAME OF TICK-TACK-TOE

By HARRY D. RUDERMAN
High School of Aviation Trades, New York, N. Y.

THE Game of *Tick-Tack-Toe* may be the nemesis of a teacher when two or more pupils prefer it to the performance of any sort of drill or decide to divert themselves away from a dull lesson. However, should this occur in a classroom, a teacher may seize the opportunity to utilize this game for the purpose of teaching some elementary topics in probability, combinations, and simple arithmetic.

The simplest *Tick-Tack-Toe* is played by two pupils with a three-by-three celled square. A move consists in placing a selected mark in each of the cells. One pupil agrees to use the mark X, and the opponent agrees to use the mark O. Only one mark is placed in any one cell. The object of the game is to obtain a straight-line sequence of three marks of the same kind. These sequences may be horizontal, vertical, or diagonal.

As soon as such a sequence is obtained by one of the two players the game is considered over, and he who obtains the sequence first is declare a winner.

As soon as one mark is placed in a cell (see Figure 1*b*) there is a definite number of ways a winning sequence can be obtained by the player as well as by his opponent. The O-player has four possible ways of winning (see Figure 1*b*) when placing his mark in the center cell, provided the winning sequence contains this mark. The strategy of his opponent, the X-player, is to prevent him from winning, while creating a favorable situation for himself. In Figure 1*c* the X-player blocked one way for the O-player and opened one possible way for himself. In Figure 1*d* the O-player blocked his opponent and threat-

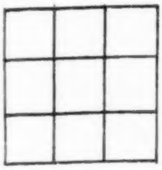


FIG. 1*a*

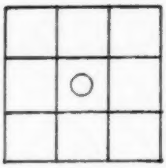


FIG. 1*b*

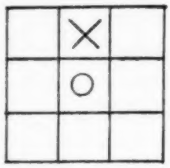


FIG. 1*c*

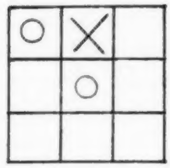


FIG. 1*d*

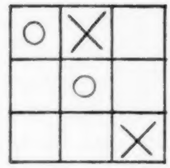


FIG. 1*e*

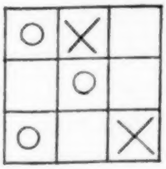


FIG. 1*f*

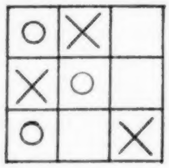


FIG. 1*g*

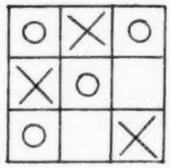


FIG. 1*h*

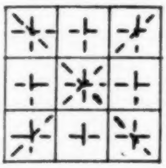


FIG. 2

3	2	3
2	4	2
3	2	3

FIG. 3

ens to win in the next move. The remainder of the game is forced.

The further analysis of the game shows that there are exactly eight possible winning sequences (see Figure 2). We shall indicate these sequences by eight straight lines. Within every cell of Figure 2 a certain number of straight lines intersect. Each straight line represents a winning pattern. Counting the number of these intersecting straight lines within each and every cell we obtain the *weight-value* of the respective cell as shown below (see Figure 3).

The strategy of playing this game must take into consideration the *weight-value* of the vacant cell which is utilized in any given move. Thus, at the beginning of the game the most preferable cell is the central one whose *weight-value* is 4. And the player who is allowed the first move should place his mark there. This gives him four different possible ways of winning. His opponent cannot block him in all the four ways during the next move. Whatever move he makes, he cannot secure for himself more than 2 possible ways to win, while he reduces his opponent's advantage to 3.

All further moves should be examined in a similar manner.

In other words, the simple *Tick-Tack-Toe* game may be very useful in the processes of analysis. Furthermore, this game offers opportunities for variation if the game is extended to squares with a greater number of cells.

The above contribution by Mr. Harry D. Ruderman, President of the Association of the Teachers of Mathematics of New York City, on the *Tick-Tack-Toe* game may appear to be too elementary for some of the teachers of mathematics. However, this department urged Mr. Ruderman to present a discussion on this topic because almost every game which is played by young or adults has a mathematical theory. The knowledge of this theory will generally improve the mastery of a game. On the other hand, many a classroom situation which begins to border on deterioration may be turned into a useful activity . . . "if only we knew how!" It is the hope of this department that the readers of THE MATHEMATICS TEACHER who know of some other games will send their contributions for publication. Mr. Ruderman informed this department that his interest in the game of *Tick-Tack-Toe* originated in a class while in service with the Armed Forces. The instructor in the class was weak on the motivation side. Thus, Mr. Ruderman's neighbor offered to teach him the three-dimensional *Tick-Tack-Toe* game. After a few minutes of instruction, Mr. Ruderman noted the mathematical properties of the game. From that moment on his friend never had a chance to beat him. This department hopes to publish the theory of the three-dimensional *Tick-Tack-Toe* game in one of the subsequent issues of THE MATHEMATICS TEACHER.

Let us consider a five-by-five *Tick-Tack-Toe* game with the rules slightly modified. In this game *all* the 25 cells must be filled with X's and O's. While the game is played a score is kept. Three like marks in a straight line and not separated by the opposing mark is given a score of 1. Four like marks in a straight line and not separated by the opposing mark is given a score of 2. Five like marks in a straight line and not separated by the opposing mark is given a score of 3. Thus, if three consecutive O's appear, the O-score is 1. If during the play this sequence of marks is increased by another O-mark, then the O-score is increased by 1, and so on.

The *weight-values* of the various cells in the five-by-five square are obtained in the same manner as in the case of the three-by-three square. These *weight-values* are shown in Figure 4 below.

9	10	11	10	9
10	15	17	15	10
11	17	24	17	11
10	15	17	15	10
9	10	11	10	9

FIG. 4

For example, the central cell has a *weight-value* 24. The reader will note that horizontally we can obtain three sequences with three consecutive marks, two sequences with four consecutive marks, and one sequence with five consecutive marks. Thus, horizontally we can obtain six scoring sequences. Vertically we can obtain six scoring sequences. Diagonally we obtain two six scoring sequences. Thus, the total number of the scoring sequences for the central cell is 24. The *weight-values* of all the other cells of the square are obtained in a similar manner.

A completed game (won by the X's) is shown in Figure 5.

Editorial Comment. In the case of the five-by-five square *weight-values* may be

O	O	O	O	X
O	X	X	X	O
O	X	X	X	X
O	O	X	O	X
O	X	X	X	O

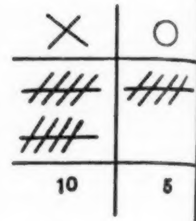
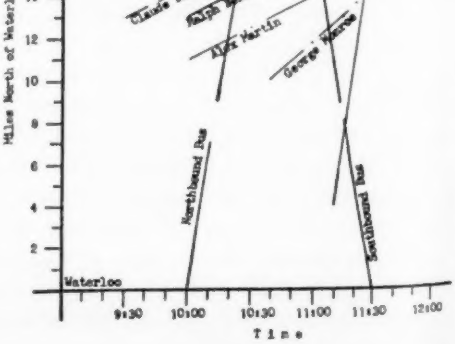


FIG. 5

also obtained in terms of the scores which are possible for any given cell. Thus, for example, in the case of the central cell, horizontally, we have three possible sequences whose scores are 1 each, two possible sequences (of four consecutive marks) whose scores are 2 each, and one possible sequence (of five consecutive marks) whose score is 3. Thus, horizontally, the *score weight-value* is 10. The vertical *score weight-value* is also 10. Diagonally we obtain two *score weight-values* of 10 each. Thus, the *score weight-value* of the central cell is 40. The computation of the *score weight-values* follows the same pattern as the computation of statistical scores with corresponding frequencies.

The above variation of the game need not be necessarily confined to squares with



the same number of cells horizontally and vertically. We may vary the game so that rectangles, or any other geometric figures may be employed.

A. B.

Here is the solution of the theft of MRS. MURPHY's laundry. The information, which was gathered by Detective Foams (see the March 1951 issue of THE MATHEMATICS TEACHER), is represented in the graph on p. 346. The only possible culprit was Ralph Bennington.

Correction: In the March issue of THE MATHEMATICS TEACHER (page 203) problem 2 should read

$$\left(\frac{5}{8}\right)^2 + \frac{3}{8} \quad \text{as} \left(\frac{3}{8}\right)^2 + \frac{5}{8}$$

BOOK SECTION

Edited by JOSEPH STIPANOWICH

Western Illinois State College, Macomb, Illinois

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is available whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

JUNIOR HIGH SCHOOL

Everyday Arithmetic (Junior Book 1), by Harl R. Douglass, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Principal, Ridge Road School, Parma, Ohio. Cloth, ix+488 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.08.

Everyday Arithmetic, (Junior Book 2) by Harl R. Douglass, University of Colorado; Lucien B. Kinney, Stanford University; and Donald W. Lentz, Principal, Ridge Road School, Parma, Ohio. Cloth, ix+502 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.08.

HIGH SCHOOL

1. Algebra

Algebra by Visual Aids (4 Volumes and Answer Book), by G. Patrick Meredith and edited by Lancelot Hogben. Cloth, 550 pages, 1948. George Allen and Unwin Ltd., Ruskin House, Museum Street, London. 10 Shillings; 8 Shillings, 6 Pence; 7 Shillings, 6 Pence; 9 Shillings, 6 Pence; 6 Shillings.

2. Plane Geometry

Plane Geometry, Revised Edition, by F. M. Morgan, Clark School, Hanover, N. H.; and W. E. Breckenridge, Teachers College, Columbia University. Cloth, viii+520 pages, 1951. Houghton Mifflin Company, 2 Park Street, Boston. $2.32.

3. Second Year Algebra

Algebra—Meaning and Mastery, Book Two, by Daniel W. Snader, University of Illinois. Cloth, v+500 pages, 1950. John C. Winston Company, 1010 Arch Street, Philadelphia 7, Pa. $2.60.

4. Special Courses in Mathematics

General Mathematics for the Shop, by Gilbert D. Nelson, Lincoln High School, Cleveland, Ohio; Frank C. Moore, Cleveland Public Schools; Carl Hamburger, Cleveland Public Schools; and Philip Becker, Editor, William E. Grady Vocational High School, Brooklyn, N. Y. Cloth, viii+440 pages, 1951. Houghton Mifflin Company, 2 Park Street, Boston. $2.64.

COLLEGE

1. Trigonometry

Trigonometry, by Cecil Thomas Holmes, Bowdoin College. Cloth, ix+246 pages, 1951. McGraw-Hill Book Co., 330 West 42nd Street, New York 18, N. Y. $3.00.

2. College Algebra

Essentials of College Algebra, by Joseph B. Rosenbach and Edwin A. Whitman, Carnegie Institute of Technology. Cloth, x+322 pages, 1951. Ginn and Company, Statler Building, Boston. $3.00.

3. Advanced Mathematics

Theory of Probability, by M. E. Munroe, University of Illinois. Cloth, viii+213 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $4.50.

The Kernel Function and Conformal Mapping, by Stefan Bergman. Mathematical Surveys No. 5. Cloth, vii+161 pages, 1950. American Mathe-

matical Society, 531 West 116th Street, New York 27, N. Y. $4.00.

The Elements of Mathematical Logic, by Paul C. Rosenbloom, Syracuse University. Cloth, iv+214 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $2.95.

Jacobian Elliptic Function Tables by L. M. Milne-Thomson, Royal Naval College, Greenwich, England. Cloth, xi+123 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $2.45.

The Fourier Integral and Certain of its Applications, by Norbert Wiener, Massachusetts Institute of Technology. Cloth, xi+201 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $3.95.

TEACHING OF MATHEMATICS

Secondary Mathematics, A Functional Approach for Teachers, by Howard F. Fehr, Teachers College, Columbia University. Cloth, xi+431 pages, 1951. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Massachusetts. $4.25.

The Teaching of Secondary Mathematics, by Charles H. Butler, Western Michigan College of Education; and F. Lynwood Wren, George Peabody College for Teachers. Second Edition. Cloth, xiv+550 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $4.75.

The Teaching of Mathematics, by David R. Davis, State Teachers College, Montclair, N. J. Cloth, 384 pages, 1951. Addison-Wesley Press, Inc., Cambridge 42, Massachusetts. $4.50.

The Teaching of Calculus in Schools, A Report prepared for the Mathematical Association. Cloth, iv+76 pages, 1951. G. Bell and Sons, Ltd., London.

MISCELLANEOUS

The Main Stream of Mathematics, by Edna E. Kramer. Cloth, xii+321 pages, 1951. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. $5.00.

REVIEWS

Arithmetic 6—The World of Numbers, Dale Carpenter and Dorothy Leavitt Pepper. New York, The Macmillan Co., 1950. iv+316 pp., $1.68.

This book is the sixth in the series called *The World of Numbers,* covering grades three through eight. The text contains some unusual features for books at this level. The topics are developed gradually and with meaning.

The material included for the purpose of improving the pupils' problem solving abilities is varied and interesting. The use of colorful drawings and pictorial material has made this one of the strongest features of the text. The suggested steps for solving problems are sound.

The text contains some valuable testing material. In the reviewer's opinion, the diagnostic testing program, if carried out by the teacher, should yield valuable information for organized review and re-teaching. The series of tests entitled "progress tests" contain some valuable items for measuring understanding. Chapter tests are also included. These contain computational exercises plus challenging "word problem" situations.

The authors have included some very interesting enrichment material that should prove of value in making mathematics more challenging to the bright pupil. The enrichment material also lends help to the authors' purpose of stressing meaning in arithmetic.—WILLIAM L. CARTER, Western Illinois State College (Laboratory School), Macomb, Illinois.

Arithmetic 7—The World of Numbers, Dale Carpenter and Elizabeth Cuthbertson. New York, The Macmillan Company, 1950. 332 pp., $1.68.

This textbook represents an arithmetic course for the seventh grade which has been well thought out, well graded, and well prepared. Emphasis is placed on understanding number concepts, principles, and generalizations. The word problems are well suited to the age and experience range of the average seventh grade child. They are presented in interesting social settings, illustrating the everyday usefulness of arithmetic. The orderly progressive organization of materials makes arithmetic in this book meaningful to the pupil, and provides effective guidance for the teacher.

An abundant variety of practice materials is provided to help increase his understanding and develop speed and skill. The vocabulary of numbers is carefully developed and maintained. Numerous attractive functional illustrations are found throughout the text; a special help to the eye minded pupil. All in all this book is on the right road to recapture the important traditional role that the textbook once held as a tool in teaching.—LOUIS M. STERN, Garrison Junior High School, Baltimore, Maryland.

Making Arithmetic Plain, Rose and Ruth Weber. Books I and II (workbooks). Wichita, Kansas, The McCormick-Mathers Publishing Company, 1949. Paper, 144 pp., $0.52 (each).

The books integrate the content of the textbook, the practice material, and the testing program in an attempt to satisfy the needs of the seventh and eighth grade children. The four functions of arithmetic; the informational, sociological, psychological, and computational are given consideration. Achievement tests are included at the end of each unit. Additional achievement tests and problems for practice in the fundamental operations are provided in separate booklets. A Teacher's Answer Book to facilitate scoring the exercises and tests is available for each grade.

The content of the books is good and the treatment of it, for the most part, is likewise

good. The traditional method of treating approximate measurement, however, is disappointing. The examples used to illustrate the process of cancellation in the multiplication and the division of decimals are objectionable from a teacher's view-point since they fail to show the complete division process.—L. H. WHITCRAFT, Ball State Teachers College, Muncie, Indiana.

Experimenting With Numbers (Teachers Manual), Catherine Stern. New York, Houghton Mifflin Company, 1950. 108 pp., $0.60.

This Teachers Manual is written to accompany newly designed types of number blocks, wherein the author describes what she calls *Structural Arithmetic* that employs the principle of measuring number—where comparisons are more readily understood and number relations more easily facilitated. Numerous activities and games are described in detail and can be understood by observing the diagrams and pictures in the book, but they can only be carried out with the special blocks described—and these are expensive. It is quite possible that the activities described will achieve desirable results because the approach makes use of education through the three senses—auditory, visual, and kinaesthetic.—JOSEPH J. URBANCEK, Chicago Teachers College, Chicago, Illinois.

Children Discover Arithmetic, Catherine Stern. New York, Harper and Brothers, 1949. xxiv +295 pp., $4.50.

Children Discover Arithmetic is the result of twenty years of experimentation by the author. This book is unique in its contribution to the teaching of arithmetic. The emphasis is on understanding the structure of numbers. It shows how to build the number concepts from 1–10 by using manipulative devices. The structure of whole numbers, common fractions and decimal fractions is presented in unforgettable pictures. The laboratory approach is used to gain understanding of numbers before attempting any sort of computation with the numbers. Special techniques for teaching bridging, and a variety of procedures that lead to mastery of the so-called fundamentals are included.

Teachers of kindergarten and of the first six grades will find that this book offers a new approach to the study of numbers. All facts are discovered through measurement rather than counting.

One of the most delightful parts of the book is the anecdotal records of the children's experiences at the Castle School in New York City where Dr. Stern is founder and director. The reader gains added insight on how children think about numbers by "listening in" on their conversations.—IDA MAE HEARD, Southwestern Louisiana Institute, Lafayette, Louisiana.

Algebra, Book 1, A. M. Welchons and W. R. Krickenberger. Boston, Ginn and Company, 1949. xi+580 pp., $2.12.

This first-year algebra text is one of the most carefully prepared, and therefore one of the most complete books that has come to the attention of this reviewer. Much attention has been given to the language of algebra and to formulas. Explanations are unusually complete, with emphasis on meanings and understanding, especially on such troublesome points as transposition and the square root process. There is a continuing and successful attempt made to show algebra as generalized arithmetic, and to integrate the subject matter with geometry, trigonometry, and analytics. The historical data inclosed is excellent as are the frequent references to the use of mathematics in the engineering and scientific world. Most teachers will particularly appreciate the unusually large number of problems of all types carefully graded into A, B, and C levels of difficulty. There are adequate tests and review materials provided throughout the text, a chapter reviewing the subject matter by topics, and a last chapter on arithmetic drill for those students particularly weak in that field. The text is well indexed and provides all the mathematical tables necessary for the work of the course. An answer book is not inclosed, but is available separately.—ROBERT V. BELDING, Thomas Carr Howe High School, Indianapolis, Indiana.

Algebra, Its Big Ideas and Basic Skills, Daymond J. Aiken and Kenneth B. Henderson. New York, Harper and Brothers, Publishers, 1950. xv+409 pp., $2.48.

Materials within this text are based upon recommendations of The National Council of Teachers of Mathematics and recent research pertaining to important concepts and processes needed for the study of advanced mathematics and college science. Creating interests that will encourage students to acquire greater understanding is of primary importance in this book. Many illustrations and photographs are included as incentives toward greater efforts. Novel caricatures are inserted for purposes of explaining mathematical processes stressing important facts and advancing thought provoking questions. An introduction addressed to the student helps initiate the program of interest creation. The reader is continually informed about algebraic applications in science and advanced mathematics.

Seven chapters are devoted to a presentation of "Big Ideas" and include discussions concerning general numbers, equations, signed numbers, dependence and mathematical operations, graphical representations of algebraic quantities, exponents at work, and indirect measurement. The other six chapters envolve further study and integration of these ideas. A breakdown of the individual chapters discloses the following presentations: more exercises than are needed permit application and review of basic principles; a section devoted to a practice of valuable skills; unit tests and a summary of things to remember. The summary will be especially valuable for teachers desiring a quick resume of a

chapter for purposes of pretesting, teaching and achievement testing. A unit entitled supplementary topics enables additional instruction in quadratics, imaginary roots and irrational numbers.—RODERICK McLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

General Mathematics, Virgil S. Mallory and Kenneth C. Skeen. Chicago, Benjamin H. Sanborn and Company, 1951. v+478 pp.

This text, according to the authors, is designed for the large majority of high school students who need that socially useful mathematics which will give them a preparation for competent and happy citizenship.

Various methods are used to motivate the work and most of the problems are within the sphere of interest of an average ninth-grade student. Each chapter ends with a test covering all work up-to-date; in other words, the test after Chapter 8 is called a Test on Chapter 1–8.

A chapter known as Practice in Computation is included toward the end of the book. This is written so that it may either be used by a student individually or as a class group. The answers to this chapter are included in the book.

The format is good, type is large, and the many illustrations and diagrams are extremely well done.—MADELINE MESSNER, Abraham Clark High School, Roselle, New Jersey.

Self-Help General Mathematics Workbook, G. E. Hawkins and L. S. Walker. Chicago, Scott, Foresman and Company, 1949. 80 pp., $0.72.

This workbook, which can be used to supplement any standard text in general mathematics, is designed, as the title indicates, to provide "self-help" to the individual pupil in a diagnostic, remedial and continuous program.

The workbook contains thirty separate cumulative drills or tests, composed of twenty examples each, based on a number of different topics. Some examples are computational, some multiple-choice, and others true or false. Each drill contains helpful illustrations and diagrams. Space is provided on the page for working the computational exercises. A progress chart provides the self-competing motivation from drill to drill.

Perhaps the most outstanding feature of this workbook is the self-help chart accompanying each of the thirty drills which enables the pupil to determine just what he needs to study in his text and workbook to correct and overcome his own individual weaknesses. There are, also, five "self-help" study units, one after every six drills, which cover the hardest examples in the preceding drills as found by the authors through the standardization of these tests.

This workbook is an excellent addition to "self-help" type of material which not only aids the pupil to locate his specific difficulties in computations, ideas and concepts, but it, also, provides a remedy and an opportunity for him to overcome his weaknesses.—HARRY L. McCULLOUGH, Woodrow Wilson Junior High School, Terre Haute, Indiana.

Algebra, Book II, A. M. Welchons and W. R. Krickenberger. Boston, Ginn and Company, 1949. x+516 pp., $2.20.

This book, designed for the second year's study of algebra is, like the first year book, unusually complete. In fact it contains most of the topics normally given in a first year college course lacking, however, a study of least squares, partial fractions, infinite series and mathematics of finance. Treatment of other topics is most excellent. Explanations are clear and well illustrated. Chapter and cumulative tests are included. There is adequate review of first year fundamentals, most of which is well integrated with the advanced work. Individual differences are cared for through grading of exercises into A and B levels of difficulty. Techniques have been emphasized, but only after concepts are thoroughly developed. Without labeling it as such, an excellent chapter on the calculus has been included. Necessary tables are included. Answers are available under separate cover. Applications of mathematics to various fields of work, historical data and other motivation materials are well done. They are appropriate, and should appeal to both student and teacher. —ROBERT V. BELDING, Thomas Carr Howe High School, Indianapolis, Indiana.

Essentials of Plane Trigonometry. Joseph B. Rosenbach, Edwin A. Whitman, and David Moskovitz. Boston, Ginn and Company, 1950. vii+158 pp., $2.70.

This book was written for early college or engineering school level, but because of its many outstanding features may be used by high school students.

The material is presented in a clear but concise manner. Each new principle or process is carefully and completely presented with all the necessary definitions, theorems, and proofs. This is followed by many illustrations, illustrative examples, and graded problems.

Other outstanding points about the book are:

(1) Historical notes are woven into the discussion. More than twenty are found in the book.

(2) The authors have tried to analyze pupil difficulties by placing *notes* throughout the book.

(3) Where misunderstanding might occur the authors have placed *warning* paragraphs.

(4) Problems are chosen from a wide subject matter field.

(5) Tables are thorough and complete.

(6) Authors suggest need for certain review before some topics are undertaken.

(7) Complete grouping of all the necessary formulas.

(8) General exercises listed by chapters and not as a composite group.—MRS. CECIL CRUM, Rossville High School, Rossville, Indiana.

A Course in the Slide Rule and Logarithms (Revised Edition), E. Justin Hills, Boston, Ginn and Company, 1950. iv+108 pp., $1.40.

This book is intended to introduce the student to the slide rule, to logarithms, to show the relationship between slide rule and logarithm procedures, and to give basic and general applications.

There are cuts of eighteen standard and special-use slide rules. Hence, the student is made aware not only of the demonstration slide rule in the class room, or the one he may possess, but also of many additional rules in existence.

Presented in twenty-two short units, each is suitable for a class period of study or for study by pupils on an individual basis. Explanations are clear and concise. Sample problems permit the student to follow through step-by-step settings on the slide rule. Adequate additional problems are given for practice, and sufficient answers are available to permit self-measurement.

Additional features include the trigonometric solution of triangles by both slide rule and five-place logarithms and the introduction to the student of slide rule scales based on natural logarithms.

The application to business, finance, and statistics which the text presents as well as basic computation problems can serve as a motivating device in appropriate secondary mathematics courses.—Russell L. Schneider, Eastern High School, Lansing, Michigan.

Business Mathematics (Third Edition), Cleon C. Richtmeyer and Judson W. Foust. New York, McGraw-Hill Book Company, Inc., 1950. xviii+441 pp., $3.50.

A successful college textbook designed primarily for students preparing to teach commercial arithmetic and students planning to enter business. Special attention has been given to development of formulae used in solving more advanced problems. After considering the usual elementary material the authors have included chapters on logarithms and statistics.

Changes in the third edition include revised exercise and problem lists, an additional chapter on statistics, and the set of miscellaneous problems has been expanded and placed in a separate chapter.—Caroline A. Lester, New York State College for Teachers, Albany, New York.

Elements of Analytic Geometry (Third Edition), Clyde E. Love. New York, The Macmillan Company, 1950. xii+218 pp., $2.75.

This is the third edition of a well-known text. It includes all the topics usually found in a course in plane analytic geometry plus chapters on coordinates in space, surfaces and curves, the plane, the straight line (in space), and quadric surfaces.

Topics that are treated more completely than in most texts include algebraic curves, asymptotes, bending of beams, rate of change of a function, trigonometric functions, exponential functions, logarithmic functions, families of curves, and the drawing of figures in three-dimensional space.

There is an abundance of illustrative examples and figures, and a wide range of exercises (about 1700), providing sufficient material so that the instructor may vary his assignments from semester to semester.

The figures are well drawn, and the type is easy to read, although the reviewer would have preferred a "glossier" grade of paper.

As a whole the book is a worthy successor to those which have preceded it.—Cleon C. Richtmeyer, Central Michigan College, Mount Pleasant, Michigan.

Strategy in Poker, Business and War, John McDonald. New York, W. W. Norton and Company, Inc., 1950. 128 pp., $2.50.

As indicated by the title, the theme of this little book is *strategy*—not just military strategy, but strategy in its multifarious applications to the affairs of mankind.

Uncertainty arises from imperfect information. Strategy is used either to clarify, or to further obscure this information. A certain kind of strategy is used to resolve conflicting desires in such a way that an optimum condition is achieved.

Part One deals with the game Poker, including an interesting account of the history of the game since its inception in America over one hundred years ago.

Part Two takes up the Theory of Games, following the general treatment in von Neumann and Morgenstern's treatise. Here we are introduced to the distinction between games of chance and games of strategy. Solitaire is strictly a game of chance, for example, since all voluntary moves are made by the single player. In games involving strategy the players have a choice of action, and endeavor to base that choice on the actions of the other players. Hence there is an interdependence, but not cooperation involved. The important principle of "minimax" is discussed at some length.

Part Three has to do with the Game of Business. The gist of this chapter is that free competition among many individuals has been replaced by strategical combinations of individuals into corporations, trade unions, and other associations. The buyer-seller relation in an open market is visualized as a two-man game wherein we again see the minimax principle at work.

The entrance of a third party into the game makes for considerable alternation, since now the possibility of coalition emerges. Viewing economic movement as a game, the author says: "The complications that one extra player adds to game solutions are both good and bad for the theory of games. For although the extra player reveals the law of coalitions, by the same token he shows the fantastic complications involved in extending actual mathematical computations to situations involving large

numbers." But the author then explains that other factors tend to resolve the situation back into a few-party game: "The trade union movement shows how large numbers of economic individuals can group themselves back into small numbers of strategy-minded units."

In Part Four the author applies the three principal features of game theory—randomized strategy, minimax strategy, and coalitions—to political and military situations. The author's discussion of Stalin's rise to power, as a lesson in coalition strategy, is extremely interesting, as is also his brief description of some of the activities of the Operations Evaluation Group in our Navy.

The material contained in this book is a development of articles originally written for *Fortune* magazine. The author's very entertaining style, supplemented by Robert Osborn's piquant illustrations, make for an enjoyable reading experience.—E. W. BANHAGEL, Northwestern University, Evanston, Illinois.

The Problem of Problem-Solving in Mathematical Instruction. Aaron Bakst. New York, New York University Bookstore, 1950. ii+53 pp., $0.40.

In this essay, prepared for teachers of mathematics and teachers of teachers, the author has undertaken a detailed analysis of the problem of problem-solving in the light of modern mathematical theories as well as the theories of learning. Not only does he examine the question in view of the purely mathematical aspects, but also in light of the psychological implications, the nature, and the inner structure of mathematical problem-situations. Attention is given to the definition of a problem; the criteria for a real problem; and the kind of mathematical problems. The author stresses the fact that unless the pupil can and does identify himself with a problem-situation, it is not real to him.

The resolution of a problem-situation, in general, is considered under the following fundamental steps:

1. Anatomy of a problem-situation
2. Transition from a problem-situation to a problem
3. Anatomy of a problem
4. Analysis of the facts and of the conditions of the problem
5. Critical evaluation of the problem
6. Plan of the solution procedure
7. Evaluation of the plan of the solution procedure
8. Analysis of the solution.

The author clarifies and shows how these steps may be applied in the resolution of a problem-situation by solving a verbal (work) problem in algebra, and an original proposition in geometry.

Unique in this field, the work provides the basis for further study and experimentation. However, the technical nature of the discussion and the author's involved mode of expression make it rather difficult reading for the average teacher of secondary mathematics.—ELIZABETH ZEIGEL WINTER, Delta State Teachers College, Cleveland, Mississippi.

Mathematics, Queen and Servant of Science, by E. T. Bell. New York, McGraw-Hill Book Co., 1951. xx+437 pages, $5.00.

This book is an integration and amplification of two earlier volumes by the same author: *The Queen of the Sciences* (1931) and *The Handmaiden of the Sciences* (1937). All the material of the two previous volumes has been retained and skillfully blended into one presentation. New material has been added to the extent of about a third of the present volume. The newer portions retain all the flavor which permeated the two originals. For those well acquainted with the earlier volumes further recommendations are hardly necessary.

The avowed purpose of this book is twofold: to give those non-mathematicians who remember enough of their elementary mathematics a better understanding of what modern mathematics is all about, and, secondly, to enable young students to catch a glimpse of modern mathematics. Certainly there is nothing more disquieting to a mathematician than to find some person holding the opinion that a mathematician is a sort of super accountant who is a whizz at calculating. This book could do much to dispel such a misconception. It would be especially suitable for collateral reading in the last year of high school or the first years of college for all students who hope to have some understanding of what mathematics is about and how it serves science.

The theme of the book is indicated by its title. It is shown how pure mathematics, the Queen, has been of great significance to science, and how, in turn, science, in using applied mathematics, the Servant, has given impetus to pure mathematics. In short, it is shown that the two have been and still continue to be inseparable. The ideas and the spirit of mathematics as it has developed from ancient times to the present day are clearly presented by Dr. Bell in a fascinating story. Interesting bits of information on the history of mathematics and mathematicians are skillfully interspersed. And many priceless gems in the form of anecdotes and witticisms are thrown in for good measure.

Chapter I is concerned with the object of mathematics and is followed by a discussion of definitions of mathematics and of the postulational method. The discussion leads up to Chapter 5, entitled "The art of abstraction," in which the basic concepts of modern algebra are presented. This chapter is completely new. In Chapter 7 geometry takes over and the author traces its development from the ancient Greeks on to relativity theory in Chapter 10. Included in Chapter 8 is a brief presentation of the basic ideas of topology. This is also new. Following Chapter 11, which is on number theory, the author turns to mathematics, the Servant of science. Following a stimulating

philosophical discussion the reader is led through the story of Kepler and Newton on to the calculus and a brief (but adequate for the purposes of the book) mention of chief topics of modern analysis. Chapter 18 is devoted to statistics and probability. The volume concludes with two chapters on 'infinity' as a mathematical term and the related questions concerning the foundations of mathematics.

This book belongs in the library of every teacher of mathematics and every serious student of science. To both it could not help but be a source of inspiration and enlightenment, for Dr. Bell has the gift of presenting the profound concepts of the subject with a minimum of technical difficulty. It has been remarked that Dr. Bell is "perhaps mathematics' greatest interpreter." There is no perhaps about it. —D. H. POTTS, Northwestern University, Evanston, Illinois.

ATTENDANCE RECORD OF THE TENTH SUMMER MEETING UNIVERSITY OF WISCONSIN, MADISON, WISCONSIN AUGUST 20-24, 1950

Compiled by EDWIN W. SCHREIBER,
Western Illinois State College, Macomb, Illinois

State	Members	Guests	Total
Alabama	2	0	2
Arkansas	2	0	2
California	3	2	5
Colorado	3	3	6
Delaware	2	0	2
District of Columbia	2	0	2
Georgia	1	1	2
Illinois	59	30	89
Indiana	18	12	30
Iowa	17	19	36
Kansas	5	8	13
Louisiana	1	0	1
Maine	0	1	1
Maryland	1	1	2
Massachusetts	1	0	1
Michigan	13	6	19
Minnesota	12	12	24
Mississippi	2	0	2
Missouri	5	0	5
Montana	1	0	1
Nebraska	5	3	8
New Hampshire	1	0	1
New Jersey	6	14	20
New York	5	12	17
North Dakota	0	1	1
Ohio	9	1	10
Oklahoma	6	5	11
Tennessee	2	1	3
Texas	5	10	15
Vermont	1	1	2
Virginia	1	0	1
West Virginia	1	1	2
Wisconsin	77	105	182
Totals	269	249	518

NEWS NOTES

An Exhibit of Instructional Aids and Demonstration Materials will be one of the features of the 1951 Summer Meeting of the National Council of Teachers of Mathematics to be held at St. Olaf College, Northfield, Minnesota. If you have materials which you would like to display, please write to Emil J. Berger, Monroe High School, St. Paul, Minnesota and let him know how much space to reserve for you. Materials may be shipped any time after May 15 to Mr. Arthur K. Solum, St. Olaf College, Northfield, Minnesota and will be stored until the time of the meeting in August.

Dates of Fall Meetings of Affiliated Groups are as follows:

Illinois Council of Teachers of Mathematics, October 13, 1951, at Northern Illinois State Teachers College at De Kalb

Iowa Association of Mathematics Teachers: Joint Meeting with Iowa State Educational Association, November, Des Moines

Oklahoma Council of Teachers of Mathematics: October 12, 1951, Oklahoma City

Mathematics Section, Eastern Tennessee Education Association: October 26, University of Tennessee, Knoxville.

A Joint Meeting of the **Kansas Association of Teachers of Mathematics** and the Kansas Section of the Mathematical Association of America was held at the University of Kansas at Lawrence on April 7. The program included the following: "Some Novel Methods for Estimating the Eigenvalues of Linear Operators" by Dr. John H. Curtiss, Chief of the National Applied Mathematics Laboratories of the National Bureau of Standards; "The Mathematical Needs of College Freshmen" by Kathleen O'Donnell of the University of Kansas; and "Motivation Devices" by W. V. Unruh of the Shawnee Mission High School.

The Mathematics Division of the Alabama Education Association held its spring meeting on March 15, 1951. **Mary E. Potter,** Supervisor of Mathematics in the Racine, Wisconsin Public Schools was the principal speaker.

(Continued on page 364)

Program

Eleventh Summer Meeting
The National Council of Teachers of Mathematics

St. Olaf College, Northfield, Minnesota
August 19, 20, 21, 22, 23, 1951

SUNDAY, AUGUST 19

3:00 P.M.–8:00 P.M. Registration and Room Assignment—Library Lobby

MONDAY, AUGUST 20

General Session

9:30 A.M.–10:30 A.M.—Gymnasium

Presiding: H. W. CHARLESWORTH, President, National Council of Teachers of Mathematics

Meeting the Needs of the Present Emergency, WILLARD E. GIVENS, Executive Secretary, National Education Association, Washington, D. C.

Study Groups

10:45 A.M.–12:00 NOON

Each study group will meet at this time each day of the conference: MONDAY, TUESDAY and WEDNESDAY. Register for the group of your choice in advance since enrollment is limited.

Group S-1—Library 201

Theme: *Mobilizing Secondary and College Mathematics Education for the National Emergency*

Leader: VERYL SCHULT, Public Schools, Washington, D. C.

Recorder: VIRGINIA LEE PRATT, Public Schools, Omaha, Nebraska

Monday's Topic: *Mobilization and the Mathematics Curriculum*

Group S-2—Holland 501

Theme: *Applications of Mathematics*

Leader: AARON BAKST, New York University, New York, N. Y.

Recorder: MAUDE HOLDEN, Public Schools, Ord, Nebraska

Monday's Topic: *Graphical Methods*

Group S-3—Library 302

Theme: *Training Teachers of College Mathematics*

Leader: JAMES H. ZANT, Oklahoma A. and M. College, Stillwater, Oklahoma

Recorder: EDWIN W. SCHREIBER, Western Illinois State College, Macomb, Illinois

Monday's Topic: *The Kind of Mathematics Preparation Needed*

Group S-4—Holland 412

Theme: *Psychological and Semantical Foundations of Meaningful Arithmetic Instruction*

Leader: HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa

Recorder: GLENADINE GIBB, Iowa State Teachers College, Cedar Falls, Iowa

Monday's Topic: *What Do We Know about How Children Learn Arithmetic?*

Group S-5—Holland 415

Theme: *Problem Solving*

Leader: PHILIP PEAK, Indiana University, Bloomington, Indiana

Recorder: HENRIETTE L. BRUDOS, State Teachers College, Valley City, North Dakota

Monday's Theme: *Objectives of Problem Solving*

Participants: JOHN RIDENOUR, Rockford, Illinois, and NAOMI MARGARET HICKS, Boone, Iowa

Group S-6—Holland 414

Theme: *Problems in Teaching Geometry*

Leader: F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

Recorder: AGNES HERBERT, Public Schools, Baltimore, Maryland

Monday's Topic: *The Purpose of Geometry in the School Program*

Group S-7—Library 202

Theme: *Problems in Teaching General Mathematics*

Leader: Myron F. Rosskopf, Syracuse University, Syracuse, New York
Recorder: Alice M. Hach, Slauson Junior High School, Ann Arbor, Michigan
Monday's Topic: *The Use of Models in the Teaching of General Mathematics*

Group S-8—Main 4
Theme: *Teaching Selected Topics of Algebra*
Leader: Daniel W. Snader, University of Illinois, Urbana, Illinois
Recorder: Margaret Joseph, Shorewood High School, Milwaukee, Wisconsin
Monday's Topic: *Teaching Solutions of Equations and Formulas by Inverse Operations*

Group S-9—Main 5
Theme: *Problems in Teaching Non-College Preparatory Mathematics in the Senior High School*
Leader: Howard F. Fehr, Columbia University, Teachers College, New York, N. Y.
Recorder: To be appointed from the group
Monday's Topic: *Who Are Our Non-College Preparatory Students of Mathematics?*

Group S-10—Library 304
Theme: *Materials of Instruction*
Leader: Henry W. Syer, Boston University, Boston, Massachusetts
Recorder: Ruth Knaus, High School, Waseca, Minnesota
Monday's Topic: *The Relationships between Objectives, Materials and Activities*

Group S-11—Main 2
Theme: *Providing for Individual Differences*
Leader: Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin
Recorder: Mary Lee Foster, Henderson State Teachers College, Arkadelphia, Arkansas
Monday's Topic: *Some Basic Differences in Slow and Fast Learners*

Discussion Groups

10:45 a.m.–12:00 Noon (Monday only)

Register for the group of your choice in advance since enrollment is limited.

Group D-11—Annex 5
Topic: *Ways of Solving Algebra Problems*
Leader: Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota

Group D-12—Annex 13
Topic: *Student-Teacher Planning*
Leader: Gladys W. Junker, University of Chicago Laboratory School, Chicago, Illinois

Group D-13—Main 13
Topic: *Remedial Mathematics at the Twelfth Grade Level*
Leader: John T. Ladd, Highland Park High School, Highland Park, Michigan

Group D-14—Annex 6
Topic: *Family Trees and Similar Devices in the Teaching of Mathematics*
Leader: Henry Swain, New Trier Township High School, Winnetka, Illinois

Group D-15—Annex 1
Topic: *Some Trends in College General Mathematics*
Leader: Lyle J. Dixon, University of Kansas, Lawrence, Kansas

Group D-16—Main 8
Topic: *How to Organize and Run a Mathematics Clinic*
Leader: Nathan Lazar, Ohio State University, Columbus, Ohio

Films and Filmstrips

10:45 a.m.–12:00 Noon—Holland 211
Arithmetic Films and Filmstrips

Sectional Meetings

1:30 p.m.–2:45 p.m. *General Section*—Library 304
Sponsored by Iowa Association of Mathematics Teachers
Presiding: Orville A. George, Mason City, President, Iowa Association of Mathematics Teachers
A Career as an Actuary, Floyd

S. HARPER, Drake University, Des Moines, Iowa

Mathematical Values of High School Freshman Exploration Days and Senior Career Days, ORLANDO C. KRIEDER, Iowa State Teachers College, Cedar Falls, Iowa

Mathematical Forms Found in Nature, KATHRYN B. HINSENBROCK, High School, Charles City, Iowa

1:30 P.M.–2:45 P.M. *General Section*—Library 302

Sponsored by Nebraska Council of Mathematics Teachers

Presiding: MILTON W. BECKMAN, President, Nebraska Section, National Council of Teachers of Mathematics

Consumer Mathematics in the Twelfth Grade, RICHARD R. SHORT, Public Schools, Lincoln, Nebraska

Creating and Maintaining Interest in Mathematics Classes. DUANE M. PERRY, Public Schools, Omaha, Nebraska

The Relative Merits of Algebra and General Mathematics in the Development of Mathematical Literacy. MILTON W. BECKMAN, University of Nebraska, Lincoln, Nebraska

1:30 P.M.–2:45 P.M. *General Section*—Holland 415

Presiding: LOUISE KINN, Junior High School, Brainerd, Minnesota

Some Implications of the Formula $K=f(d)$, where d=discovery and K=knowledge, for Teachers of Mathematics. LEONARD S. LAWS, University of Minnesota, Minneapolis, Minnesota

General Educational Values of Mathematics and the Attempt of a Faculty to Teach Them. JOHN ABERNATHY, Arkansas Polytechnic College, Russelville, Arkansas

1:30 P.M.–2:45 P.M. *Arithmetic Section*—Steensland Auditorium

Presiding: HARRY C. JOHNSON, University of Minnesota, Duluth, Minnesota

Some Fun With Arithmetic. HAROLD D. LARSEN, Albion College, Albion, Michigan

Problem Solving, The Backbone of Arithmetic. HOLMES BOYNTON, Northern Michigan College of Education, Marquette, Michigan

1:30 P.M.–2:45 P.M. *General Mathematics*—Holland 501

Presiding: PAUL JORGENSON, High School, Northfield, Minnesota

The Possibility and Desirability of Teaching Arithmetical Concepts and Operations without the Use of Numerals and Algorisms. NATHAN LAZAR, Ohio State University, Columbus, Ohio

Teaching General Mathematics Effectively. GILBERT D. NELSON, Lincoln High School, Cleveland, Ohio

1:30 P.M.–2:45 P.M. *Algebra Section*—Library 303

Presiding: WORTH J. OSBURN, University of Washington, Seattle, Washington

Getting to the Heart of Algebra. DANIEL W. SNADER, University of Illinois, Urbana, Illinois

The Function Concept in Secondary School Mathematics. J. H. BANKS, George Peabody College for Teachers, Nashville, Tennessee

1:30 P.M.–2:45 P.M. *Geometry Section*—Library 201

Presiding: THEODORE KELLOGG, President, Mathematics Section, Minnesota Education Association, Minneapolis, Minnesota

The Nature of Evidence. HOWARD F. FEHR, Teachers College, Columbia University, New York, N. Y.

The Mult-Converse Concept in Geometry. FRANK B. ALLEN, High School, LaGrange, Illinois

1:30 P.M.–2:45 P.M. *College Section*—Library 202

Presiding: CLARENCE S. CARLSON, St. Olaf College, Northfield, Minnesota

A Combined Course in College Algebra and Trigonometry. JAMES H. ZANT, Oklahoma A. and M. College, Stillwater, Oklahoma

Courses Desirable for Training Teachers of High School Mathematics. CECIL B

READ, University of Wichita, Wichita, Kansas

Films and Filmstrips

1:30 P.M.–2:45 P.M.—Holland 211
Arithmetic Films and Filmstrips

Coffee Hour

3:00 P.M.–4:00 P.M.—College Center

Laboratory Sessions

3:30 P.M.–4:30 P.M.

Each laboratory group will meet at this time each day of the conference: MONDAY, TUESDAY, and WEDNESDAY. Register for the laboratory of your choice since enrollment is limited to 50. There may be a small charge for the materials which are used in the making of teaching devices by the members of each group.

Arithmetic Laboratory Section—Library 304
Leader: ELDA MERTON, Chicago, Illinois

Junior High School Laboratory Section—Library 303
Leader: MARVIN JOHNSON, Junior High School, Rochester, Minnesota

Senior High School Laboratory Section—Library 301
Leader: EMIL J. BERGER, Monroe High School, St. Paul, Minnesota
Participant: FRANKLYN BLUME, Principal, Monroe High School, St. Paul, Minnesota
Topic: *The Role of the Administration in Developing a Mathematics Laboratory*

Film and Filmstrip Exhibit

3:30–4:30 P.M.—Holland 211
Arithmetic Films and Filmstrips

Exhibits

9:00 A.M.–5:00 P.M.—Library Reference Room
Commercial Exhibits
Mathematical Models and Teaching Devices
Charts, Pamphlets and other Materials

TUESDAY, AUGUST 21

General Session

9:30 A.M.–10:30 A.M.—Gymnasium
Presiding: EDITH WOOLSEY, President, Minnesota Council of Teachers of Mathematics, Minneapolis, Minnesota
What Mathematics Should We Be Teaching? JOHN R. MAYOR, University of Wisconsin, Madison, Wisconsin

Study Groups

(See Monday's Sechedule)

10:45 A.M.–12:00 NOON

Group S-1—Library 201
Topic: *The Role of Mathematics in National Defense*
Leader: KENNETH HENDERSON, University of Illinois, Urbana, Illinois

Group S-2—Holland 501
Topic: *The Mathematics of Atomic Energy*

Group S-3—Library 302
Topic: *The Professional Courses Needed*

Group S-4—Holland 412
Topic: *Fundamental Arithmetic Situations*

Group S-5—Holland 415
Topic: *Characteristics of Good Problem Material*

Group S-6—Holland 414
Topic: *Functional Dependence in Geometry*

Group S-7—Library 202
Topic: *The Use of Films and Filmstrips in the Teaching of General Mathematics*

Group S-8—Main 4
Topic: *Teaching Fundamental Operations with Directed Numbers*

Group S-9—Main 5
Topic: *What Are the Best Teaching Devices for This Group?*

Group S-10—Library 304
Topic: *The Location and Evaluation of Materials*

Group S-11—Main 2
Topic: *Helpful Topics for Slow Learners*

Discussion Groups

10:45 A.M.–12:00 NOON (Tuesday only)

Group D-21—Annex 1
Topic: *What Shall We Teach in First Year Algebra*

Leader: A. M. WELCHONS, Arsenal Technical Schools, Indianapolis, Indiana

Group D-22—Annex 5

Topic: *The Emerging Extension of General Mathematics in Senior High School*

Leader: F. G. LANKFORD, JR., University of Virginia, Charlottesville, Virginia

Group D-23—Annex 13

Topic: *How Can We Best Achieve Soundness in Our Procedure, Reasoning, and Terminology*

Leader: HENRY A. MEYER, Central High School, Evansville, Indiana

Group D-24—Main 13

Topic: *Applications of Teacher-Pupil Planning in the Mathematics Classroom*

Leader: LOTTCHEN LIPP HUNTER, Curriculum Division, Wichita, Kansas

Group D-25—Main 8

Topic: *Trends in the Teaching of Solid Geometry*

Leader: WILLIAM D. POPEJOY, High School, Savanna, Illinois

Group D-26—Annex 6

Topic: *Ways and Means to Implement Thinking in Arithmetic*

Leader: ANN PETERS, Teachers College, Keene, New Hampshire

Film and Filmstrip Exhibit

10:45 A.M.–12:00 NOON—Holland 211
Junior High School Films and Filmstrips

Sectional Meetings

1:30 P.M.–2:45 P.M. *General Section*—Drama Studio

Report of Representative of Affiliated Groups

Presiding: MARY C. ROGERS, High School, Westfield, New Jersey

Significant Services of Mathematics Education

1:30 P.M.–2:45 P.M. *General Section*—Library 304

Presiding: LAWRENCE WAHLSTROM, State Teachers College, Eau Claire, Wisconsin

The Preparation of Teachers. MYRON ROSSKOPF, Syracuse University, Syracuse, New York

The Evaluation of Successful Teaching of Mathematics. PHILIP PEAK, Indiana University, Bloomington, Indiana

1:30 P.M.–2:45 P.M. *General Section*—Library 201

Presiding: ONA KRAFT, Collinwood High School, Cleveland, Ohio

Reasoning Patterns as Proof Aids. VELMA OAKES, Siloam Springs, Arkansas

Adjusting First Year College Mathematics to Meet the Needs of Special Interest Students. ADRIEN L. HESS, Montana State College, Bozeman, Montana

Mathematics in Michigan's Outdoor Education Program for High School Students. DONALD F. MARSHALL, High School, Dearborn, Michigan

1:30 P.M.–2:45 P.M. *Arithmetic Section*—Steensland Auditorium

Presiding: MARIE S. WILCOX, Washington High School, Indianapolis, Indiana

Developing Number Concepts in the Primary Grades. AGNES G. GUNDERSON, University of Wyoming, Laramie, Wyoming

Basic Meanings in Arithmetic. F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

1:30 P.M.–2:45 P.M. *General Mathematics Section*—Holland 501

Presiding: MILTON W. BECKMAN, University of Nebraska, Lincoln, Nebraska

Examples of Laboratory Teaching in Mathematics. F. G. LANKFORD, University of Virginia, Charlottesville, Virginia

Motivation as the Key. RALPH HUFFER, Beloit College, Beloit, Wisconsin

1:30 P.M.–2:45 P.M. *Algebra Section*—Library 303

Presiding: FLORENCE SHOTTLER, High School, Austin, Minnesota

The Semantics of Algebra. Worth J. Osburn, University of Washington, Seattle, Washington

Four Fundamental Processes in Mathematics. John Reckzeh, New Jersey State Teachers College, Jersey City, New Jersey

1:30 p.m.–2:45 p.m. *Geometry Section*—Library 201

Presiding: Clarice Kaasa, High School, Red Wing, Minnesota

Honor Work in Demonstrative Geometry. Eloise Bakst, Jamaica High School, New York, N. Y.

Successive Levels of Generalization in Geometry. Clarence H. Heinke, Capital University, Columbus, Ohio

1:30 p.m.–2:45 p.m. *Advanced General Mathematics Section*—Holland 415

Presiding: Jack Wells, Board of Education, Minneapolis, Minnesota

What Should Be Taught at the Twelfth Grade Level in Consumer Mathematics. Margaret Joseph, Shorewood High School, Milwaukee, Wisconsin

A Course of Study for the Second Year of General Mathematics. William Posorske, High School, Rochelle, Illinois

1:30 p.m.–2:45 p.m. *College Section*—Library 202

Panel Discussion: *The Improvement of Instruction in College Mathematics*

Participants: Max Kramer, New Mexico College of Agriculture and Mechanic Arts, Las Cruces, New Mexico; Harold D. Larsen, Albion College, Albion, Michigan; and Kenneth Wegner, Carleton College, Northfield, Minnesota

Film and Filmstrip Exhibit

1:30 p.m.–2:45 p.m.—Holland 211

Junior High School Films and Filmstrips

Coffee Hour

3:00 p.m.–4:00 p.m.—College Center

Recreational Activities

3:00 p.m.–5:00 p.m.

Laboratory Sessions

3:30 p.m.–5:30 p.m. (See Monday Schedule)

Film and Filmstrip Exhibit

3:30 p.m.–4:30 p.m.

Junior High School Films and Filmstrips

Exhibits

9:00 a.m.–5:00 p.m.—Library Reference Room

Commercial Exhibits

Mathematical Models and Teaching Devices

Charts, Pamphlets and other Materials

Dinner

6:30 p.m.

Norwegian Dinner

Entertainment

8:30 p.m.

Folk Dancing, Square Dancing, Bridge

Wednesday, August 22

General Session

9:30 a.m.–10:30 a.m.—Gymnasium

Presiding: Henry Van Engen, Iowa State Teachers College, Cedar Falls, Iowa

Making Mathematics Meaningful. Leo J. Brueckner, University of Minnesota, Minneapolis, Minnesota

Study Groups

(See Monday Schedule)

10:45 a.m.–12:00 Noon

Group S-1—Library 201

Topic: *Successful Features of the Training Program During World War II*

Leader: William L. Hart, University of Minnesota, Minneapolis, Minnesota

Group S-2—Holland 501

Topic: *The Banker's Number—e*

Group S-3—Library 302

Topic: *The Psychology of Learning Applied to Mathematics*

Group S-4—Holland 412

Topic: *Problem Solving*

Group S-5—Holland 415
Topic: *Problem Solving Applied to Life*
Group S-6—Holland 414
Topic: *What About Solid Geometry*
Group S-7—Library 202
Topic: *Sharing our Experiences in Teaching General Mathematics*
Group S-8—Main 4
Topic: *Teaching Problem Solving, or Teaching for Integration of Algebra and Geometry, or Teaching of Logarithms and the Slide Rule*
Group S-9—Main 5
Topic: *The Subject Matter and the Training of the Teachers*
Group S-10—Library 304
Topic: *The Proper Use of Materials*
Group S-11—Main 2
Topic: *Adapting Teaching to Slow Learners*

Discussion Groups

10:45 A.M.–12:00 NOON (Wednesday only)

Group D-31—Annex 5
Topic: *What Elements of Statistics Should We Teach in High School*
Leader: MAX BEBERMAN, University of Illinois High School, Urbana, Illinois
Group D-32—Annex 13
Topic: *What Belongs in Fourth Semester High School Algebra*
Leader: ONA KRAFT, Collinwood High School, Cleveland, Ohio
Group D-33—Main 13
Topic: *Junior High School Mathematics in the New Education*
Leader: MARY C. ROGERS, High School, Westfield, New Jersey
Group D-34—Main 8
Topic: *How May Plane Geometry Be Revised to Meet the Life Adjustment Education Program*
Leader: CELIA H. CANINE, North High School, Wichita, Kansas
Group D-35—Annex 1
Topic: *Ideas for Adding Interest to the Mathematics Club or Classroom*
Leader: EDNA NORSKOG, Illinois State Normal University, Normal, Illinois
Group D-36—Annex 6
Topic: *The Measurement of Understanding in Demonstrative Geometry*
Leader: JACK SILVERMAN, St. John's High School, Winnipeg, Canada

Film and Filmstrip Exhibit

10:45 A.M.–12:00 NOON—Holland 211
Geometry Films and Filmstrips

Sectional Meetings

1:30 P.M.–2:45 P.M. *General Section*—Gymnasium
Presiding: HOLMES BOYNTON, Northern Michigan College of Education, Marquette, Michigan
The Educated of the Gifted. WILBUR MURRA, Educational Policies Commission, Washington, D. C.
Human Relations in the Mathematics Classroom. H. E. BENZ, Ohio University, Athens, Ohio

1:30 P.M.–2:45 P.M. *Applications Section*—Drama Studio
Presiding: MARVIN JOHNSON, Rochester, Minnesota
Mathematics and Music. AARON BAKST, New York University, New York, N. Y.

1:30 P.M.–2:45 P.M. *General Section*—Library 304
Sponsored by the Wisconsin Council of Mathematics Teachers
Panel Discussion: *Improving the Testing Program*
Presiding: KENNETH R. FISH, President, Wisconsin Council of Mathematics Teachers, Beloit, Wisconsin
Participants: JOHN R. MAYOR, University of Wisconsin, Madison; RALPH E. HUFFER, Beloit College, Beloit; JOHN BROWN, Wisconsin High School, Madison; MARGARET STRIEGL, Senior High School, Wauwatosa; DOROTHY SWARD, Roosevelt Junior High School, Beloit; ELLI OTTESON, Senior High School, Eau Claire; IRENE LARSON, Elementary Supervisor, Green Bay; and SISTER MARY FELICE, Mount Mary College, Milwaukee

1:30 P.M.–2:45 P.M. *Arithmetic Section*—Steensland Auditorium

Presiding: LENORE JOHN, University of Chicago Laboratory School, Chicago, Illinois

Dr. Stern's Structural Arithmetic Materials. ELIZABETH BALDWIN, Kamehameha Schools, Honolulu, Hawaii

Teaching Mathematical Generalizations in the Elementary School. LUCY ROSENQUIST, Colorado State College, Greeley, Colorado

How to Make Individualized Instruction in Arithmetic Meaningful. ALICE FUNFAR, University of Iowa Graduate Student, Iowa City, Iowa

1:30 P.M.–2:45 P.M. *General Mathematics Section*—Holland 501

Presiding: GEORGE MCCUTCHEON, Minnesota State Department of Education, St. Paul, Minnesota

Teaching the Concepts of Area and Volume to Avoid Certain Pitfalls. ALICE M. HACH, Slauson Junior High School, Ann Arbor, Michigan

Dimensionality—A Proposal of a New and Important Thread of Emphasis in Secondary Mathematics and Science. SHELDON S. MYERS, Ohio State University School, Columbus, Ohio

1:30 P.M.–2:45 P.M. *Algebra Section*—Library 303

Presiding: NICK LOVDJIEFF, University of Minnesota High School, Minneapolis, Minnesota

Oral Practice Work in Mathematics. P. A. PETRIE, University of Toronto, Toronto, Ontario

Let's Return to the Fundamental Axioms. KENNETH R. CONKLING, Frankton, Indiana

1:30 P.M.–2:45 P.M. *College Section*—Library 202

Presiding: KENNETH MAY, Carleton College, Northfield, Minnesota

The Mathematics of Meteorology, MAX KRAMER, New Mexico College of Agriculture and Mechanic Arts, Las Cruces, New Mexico

The University Recreational Mathematics Lecture for High School Students. CHARLES HATFIELD, University of Minnesota, Minneapolis, Minnesota

Films and Filmstrip Exhibit

1:30 P.M.–2:45 P.M.
Geometry Films and Filmstrips

Coffee Hour

3:00 P.M.–4:00 P.M.—College Center

Laboratory Sessions

3:30 P.M.–5:30 P.M.
(See Monday Schedule)

Films and Filmstrip Exhibit

3:30 P.M.–4:30 P.M.
Geometry Films and Filmstrips

Recreational Activities

3:00 P.M.–5:00 P.M.

Exhibits

9:00 A.M.–5:00 P.M.—Library Reference Room

Commercial Exhibits

Mathematical Models and Teaching Devices

Charts, Pamphlets, and other Materials

Banquet

7:00 P.M.

Toastmaster: WILLIAM L. HART, University of Minnesota, Minneapolis, Minnesota

Mathematics, Logic and Values, DR. ROBERT H. BECK, University of Minnesota, Minneapolis, Minnesota

THURSDAY, AUGUST 23

Sight-Seeing Trip

9:00 A.M.–4:00 P.M.
Twin Cities

ANNOUNCEMENTS

Registration: The registration fee is fifty cents for members of the National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools.

The fee for non-members and visitors is \$1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press and commercial exhibitors are not charged a registration fee, but should register. You are urged to register in advance using the advance registration and reservation form.

Room Reservations: Applications for room accommodations in St. Olaf dormitories should be sent to David Johnson, Director of Public Relations, St. Olaf College, Northfield, Minnesota, by August 10. Rates are as follows: \$1.50 per person per day. All rooms are double or triple rooms, with single beds and bunks, and must be used as such. The dormitories will provide linens and towels, however no maid service is available. Rooms will be available for the period, Sunday, August 19, through Friday, August 24. Reservations for residence accommodations may be made by mailing the Advance Reservation and Registration Form, *not* later than August 10. Room charges are payable in advance at time of registration.

Food Service: Meals and coffee hours will be provided by college facilities, Monday morning through Friday morning, August 20–24, and breakfast and luncheon will be cafeteria style.

Evening Meals: The evening meals will consist of a picnic on Monday (\$1.00); Norwegian dinner on Tuesday (\$2.00); and the banquet on Wednesday (\$2.50). Reservations for evening meals should be made on the Advance Reservation and Registration Form.

Study Groups: In order to permit more intensive study of classroom problems, study groups have been organized, to meet from 10:45 to 12:00 Noon each day of the conference. If you wish to meet with a study group, registration should be made in advance on the Advance Registration and Reservation Form, since enrollment in most of the groups is limited. If you wish to be a member of a study group, you will register for the group of your choice, and attend the meetings of the group on each day of the conference. Attendance to that group will be by admission card which will be issued at the time of registration.

Discussion Groups: Discussion groups will be small to permit the audience to participate in the topic of discussion. Since enrollment is limited, registration should be made in advance with first, second, and third choices indicated. Attendance to that group will be by admission card which will be issued at the time of registration.

Mathematics Laboratories: There will be three mathematics laboratories sections, each section meeting on each day of the conference. These groups will demonstrate and discuss the construction and use of models in the classroom. Then each member of the group will be given the opportunity to make models. A minimum cost charge for materials used will be made. You are urged to register in advance since enrollment in each section is limited.

Recreational Facilities: The recreational facilities of St. Olaf College will be available to everyone attending the conference and members of their families. These facilities include swimming pool, tennis court, golf course. Special activities for wives and children in attendance will be provided each afternoon of the conference.

Sight Seeing Trip: A sight seeing trip by bus will be provided on Thursday, August 23, to points of interest in the Twin Cities. The bus will leave St. Olaf College at 9 A.M. Thursday, August 23, and return at approximately 4 P.M. The trip will include an excursion through the Ford Assembly Plant, luncheon at the Automobile Club, a tour of the Minnesota River valley, Minnehaha Falls, Minneapolis lakes, the University of Minnesota campus and the Museum of Art. The cost of bus fare for this excursion will be \$1.75. The cost of the luncheon will depend upon individual orders. Make reservations for this trip on the registration form before August 10.

Exhibits: There will be exhibits of

mathematical models, instruments, teaching aids and other classroom materials. Teachers are invited to bring materials for exhibit. Those who are willing to bring materials from their schools, should communicate in advance with Emil Berger, Monroe High School, St. Paul, Minnesota.

Many free and inexpensive materials will also be on exhibit. Suggestions of recent materials which have become available and have classroom value should be sent to Florence Schottler, High School, Albert Lea, Minnesota.

Commercial Exhibits: Textbooks and commercial teaching aids will be on exhibit. Inquiries for exhibit space should be addressed to Lawrence Goodrich, High School, Owatonna, Minnesota.

Supplies and Equipment: Speakers and other participants in the program who need projection equipment or other materials should communicate not later than August 10 with Florence Collins, 1406 Osceolo Street, St. Paul, Minnesota.

Films and Film Strips: Persons wishing to view films while in attendance at the meeting should communicate with Oscar A. Wilcox, High School, Anoka, Minnesota, not later than August 1.

Refunds: No ticket refunds will be made later than three hours preceding functions for which reservations were made, e.g., picnic, Norwegian dinner, banquet and sight seeing trip.

Information: The Hospitality Committee will furnish you with information on vacation facilities, points of interest, etc. A daily bulletin of special activities will be published. When at the conference ask for help at the Information Desk in the library lobby.

Luncheons: Luncheons will be served cafeteria style from 12 noon through 1 P.M. Special programs of a social nature will be provided.

Coffee Hour: Coffee hours will be provided each afternoon in a college center. This will provide an opportunity to meet friends and relax between sessions.

ADVANCED REGISTRATION AND RESERVATION FORM

Fill out completely and mail to Mr. David Johnson, Public Relations Office, St. Olaf College, Northfield, Minnesota, as soon as possible.

Mr, Miss, Mrs ____

First Name Initial Last Name

Address ____

Street and Number City State

Registering as: Member of NCTM ________; MAA ________; Student ________ Elementary Teacher ________ Exhibitor ________; Non-Member ________

Members of families and friends of those attending the meetings are welcome at all social and recreational events.

Registration Fee: **$.50 or $1.50** ________

Room Registration:

______ **Myself only. I wish to room with** ______________________

______ **Myself and wife or husband** ______________________

______ **Myself and the following members of my family. Give ages of children** ________

____

____

Encircle days room is desired. August 20, 21, 22, 23

See over

News Notes

(*Continued from page 353*)

Members of the **Association of Mathematics Teachers in New England** at their March 10 meeting were provided with interesting teaching materials as a result of visiting twenty exhibits consisting of a game or puzzle based on a principle of mathematics. The procedure for visiting the exhibits was governed by mathematical formulas governing the selection of different partners at each table. Typical exhibits involved playing games of three-dimensional tick-tack-toe, piecing together various Geometric Dissections and Mystic Pyramids, construction of geometric figures using the 7-pieces of a tangram set, making and solving magic squares, working with Moebius strips, and making geometric "faces." The program was supervised by quizmasters William R. Ransom of Tufts College, Henry W. Syer of Boston University, and Mrs. Isabel Savides of the Newton Public Schools.

Charles H. Butler, Professor of Mathematics at Western Michigan College at Kalamazoo, will be one of the members of the guest faculty for the 1951 Summer Session at the **Colorado State College of Education at Greeley.** Dr. Butler will participate in a workshop for mathematics teachers during the pre-session period, June 11–22 and will teach classes during the regular eight-weeks session, June 22–August 17.

New York University's Department of Mathematics Education of the School of Education is offering the following courses during its session from July 2 through August 10: The Teaching of Arithmetic, by Amanda Loughren; The Teaching of Junior High School Mathematics, by Harry Ruderman; The Teaching of Mathematics in the Senior High School, by John J. Kinsella; Teaching and Curricular Problems of College Mathematics, by A. Day Bradley; Research Investigations in Mathematics Education, by J. J. Kinsella; and Applications of Mathematics, by A. Day Bradley.

Films for Secondary Mathematics is the title of a 90 page mimeographed booklet which may be obtained for seventy-five cents from Professor Henry W. Syer, School of Education, Boston University, Boston, Massachusetts. It contains descriptions of most of the films now available as well as a summary of teachers' opinions which were obtained in a special research project at Boston University.

Mathematical Pie is the name of a new publication of the Mathematics Staff of the Gateway School, Leicester, England, consisting of puzzles and problems of special interest to students at secondary school level. The first issue which appeared in October, 1950, consists of 4 pages and is available at 1d. while the second issue of 12 pages which was published in February 1951 may be secured for 2d. from the editor R. H. Collins at the Gateway School, Leicester, England. Arrangements are being made for distribution of future issues through an agent in the United States.

Mr. A. H. Wheeler, a teacher of mathematics at Worcester High School and professor of mathematics at Clark University, died on December 19, 1950. He served as a member of the first Board of Directors of the National Council in 1920–21 and as a member of its Multi-Sensory Aids Committee in 1943–45.

President Truman has appointed 24 persons as members of the Board of Directors of the **National Science Foundation.** In addition to **Dr. Harold Marston Morse,** professor of mathematics at Princeton University, there are three physicists, four chemists, four biologists, three medical scientists, four educators, two engineers, one geologist and two industrialists on the Board.

(*Continued from reverse side*)

Evening Registration: Indicate the number of reservations desired for each

Picnic ($1.00) ______ Norwegian Dinner ($2.00) ______ Banquet ($2.50) ______

Group Registration: Be sure to state complete Group Number and leader. Remember, each study group meets *every* day of the conference.

Choice:	*Monday*	*Tuesday*	*Wednesday*
First	______	______	______
Second	______	______	______
Third	______	______	______

Laboratory Section Registration: (Register for only one section)
Each group meets Monday, Tuesday, Wednesday

Arithmetic Section ______ Junior High School ______ Senior High School ______

Enrollments in study groups, discussion groups and laboratory sections will be made on the basis of the time of registration and the number enrolled will be limited.